A UTILITY-THEORY TRAVEL DEMAND MODEL
INCORPORATING TRAVEL BUDGETS†

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Abstract—A model of traveler behavior is proposed which is consistent with the possibility that travelers
spend average daily amounts of time and money on travel with stable regularities both among urban areas
and over time in the same area. The model is founded on economic utility theory. It is designed to forecast:
(1) the amount of total travel generated by types of households, (2) the division of travel among available
modes, and (3) the relationship between the amounts of time and money allocated to travel expenditures.
The qualitative properties of the model are shown to be consistent with economic principles. Specific
theoretical results reveal that, in the simultaneous presence of constraints on both time and money, travel
budgets are not solely constant proportions of income and time available as they are in the cases of single
constraints relevant to classes of travelers to whom time is scarce compared to money, or conversely.
Constant expenditure proportions are shown to be linear approximations which are subject to empirical
validation. The relevant economic principle is that expenditures can be considered fixed in the short run but
become flexible in the long run when utility maximization is applied to the expenditures themselves and not
just to their allocation. Empirical tests of the model using data from three urban areas are positive, but
additional tests are called for. The most important output of the research is deemed to be the establishment
of theoretical hypotheses which can be used in continuing tests of travel budgets.

INTRODUCTION

It has been observed that groups of travelers within urban areas appear to have average daily
time and money expenditures on travel which display stable trends both among urban areas and
over time in the same area (Tanner, 1961; Ot and Shuldiner, 1962; Morgenstern, 1967; Szalai, 1972;
Goodwin, 1973; Zahavi, 1974, 1976 and 1979a). Such expenditures have been called travel
budgets. Various methods of employing travel budgets to improve conventional travel demand
models have been explored (e.g. Tanner, 1961; Halder, 1970; Kirby, 1974), and one new
methodology using travel budgets has been proposed as an alternative to conventional models
(Zahavi, 1977, 1979b and 1979c). Continuing research is addressing the stability and predictabil-
ity of travel budgets on individual traveler levels.

The UMOT Model developed by Zahavi (1979c) is designed to take advantage of the
regularities of travel budget expenditures to interrelate travel demand and transportation
system supply with urban form. A major motivation behind this approach involves limiting the
number and complexity of the independent variables so that predictions of future conditions
are made without having to rely solely upon independent variables which have proven to be
difficult to predict in their own right. Obviously, development of such an approach requires the
establishment of theoretical as well as empirical underpinnings. The purpose of the research
reported in this paper is to determine whether or not there is a theoretical basis for travel

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Germany Ministry of Transport under the title "The UMOT Project" (Zahavi, 1979k).
behavior models which allows the incorporation of travel budgets in a manner consistent with behavioral science principles. This research is needed to establish hypotheses for evaluating the predictabilities of budget levels and to link travel with other variables, such as household expenditures on other goods and services and allocations of time among activities (Goodwin, 1979; Wigan and Morris, 1979).

The proposed theoretical basis involves utility theory from microeconomics. Utility theory in turn is based upon the premise of rational choice behavior. Rational choice behavior asserts that a decision-maker is able to rank possible alternatives in order of personal preference and will choose that alternative that is ranked highest, subject to relevant constraints placed on the choice decision.

Axioms have been developed which define under what conditions preferences in rational choice behavior can be represented by a continuous numerical function known as a utility function (Debreu, 1954). Most rational choice scenarios involving consumer choices of goods and services, including those involving travel choices, easily pass the tests for utility representation. However, there are important exceptions. One example of a scenario outside the scope of traditional utility theory involves lexicographic ordering: a traveler might prefer the fast mode of transportation under all circumstances unless two modes are equally fast, in which case he or she prefers the cheaper of the two.

Assuming that travel preferences are transitive and continuous, rational choice can be translated into utility terms in the following manner: a traveler chooses that alternative which maximizes his or her utility, subject to relevant constraints. The arguments of the utility function can be either actual amounts of the goods and services which comprise the choice alternatives, or levels of characteristics or attributes which are produced by the goods and services in varying proportions. The former approach is that of traditional microeconomics, while the latter approach is associated with relatively recent attempts at generalizing constructs of consumer behavior (Lancaster, 1966). Both approaches have been applied to travel behavior.

Applications of utility theory to travel behavior have been reported in the contexts of mode choice (Quandt and Baumol, 1966; Shunk and Bouchard, 1970), spatial interaction, which encompasses trip generation and distribution in conventional planning processes (Niedercorn and Bechdolt, 1969; Beckmann and Golob, 1972; Golob et al., 1973), time allocations (de Donnea, 1971; Evans, 1972; Burns, 1979) and combinations of various travel decisions (Golob and Beckmann, 1970; Charles River Associates, 1972).

THE GENERAL FORM OF A UTILITY MODEL

Strotz (1957, 1959) proposed a theory of household decision making based on additive separable utilities. The theory states that consumers allocate expenditures on the various commodity groups, such as food items, housing and leisure time, so as to maximize utility subject to budget constraints. The consumer then allocates expenditures on each good and service, subject to the constraints on the total amounts allocated to the commodity groups. Small changes in prices of individual goods and services do not affect first-stage group decisions. However, substantial changes in prices which influence relative price indices of the groups feed back to the first-stage decisions, potentially leading to reallocations among the commodity groups.

A definition of commodity groups relevant to modeling travel decisions is

\[ u = u(x, c, t) \]  \hspace{1cm} (1)

where \( u \) is household utility, \( x \) is the amount or quantity of travel, \( c \) is consumption of non-travel goods and services (call this general consumption), and \( t \) is leisure time. Economic theory dictates that the function \( u \) exhibit “diminishing marginal utilities” (i.e. \( u \) is monotonically increasing and quasi-concave in the domains of the goods and services or attributes).

Specifying price indices for travel and general consumption as \( p_t \) and \( p_c \), respectively, the household faces the following money budget constraint when allocating expenditures:

\[ p_t x + p_c c \leq Y \]  \hspace{1cm} (2)
A utility-theory travel demand model incorporating travel budgets

where \( Y \) is household disposable income. Similarly, the time budget constraint is

\[
t_i x + t + t_c = \mathcal{T}
\]

(3)

where \( t_i \) is time per unit distance traveled, \( t_c \) is time for general consumption, and \( \mathcal{T} \) is the total time available to all household members. Presumably, \( \mathcal{T} \) is total clock time for the analysis period multiplied by the number of persons in the household minus time required for nondiscretionary activities such as sleep and income generation. Assuming for first approximation that time spent for general consumption is relatively constant over the range of consumption levels dealt with in the short-term, time available net of time for general consumption, \( T = \mathcal{T} - t_c \), can be used as the time constraint on the right side of inequality (3).

The first-stage decision is then to maximize (1) subject to (2) and (3). Assuming additivity, this decision becomes:

\[
\max_{x, c} \max_{t, \xi} \left[ \phi(x) + \psi(c) + \xi(t) \right]
\]

subject to all quantities being non-negative and

\[
p_c x + p_c c \leq Y
\]

(4)

\[
t_i x + t \leq T
\]

where \( \phi, \psi \) and \( \xi \) represent utilities due to travel, general consumption and leisure, respectively. The additivity assumption implies that the marginal rates of substitution among various goods and services within one commodity group are independent of the marginal rates of substitution among goods and services within any other commodity group. Nevertheless, the actual levels of consumption of any good or service within one commodity group are dependent upon the overall price index of another commodity group because all commodity groups compete for the same resources. In other words, shift in prices among various consumer products, for instance, would not be expected to affect travel expenditures unless such shifts changed the overall price index of general consumption relative to the price index of transportation.

Income not consumed or saved has no intrinsic value, and the same is true of available time not allocated to leisure or some activity. Consequently, the budgets of problem (4) can be considered binding and the inequality signs vanish (in cases where one constraint alone determines the decision, the second budget constraint can be considered redundant). Solving for consumption \( c \) and leisure time \( t \), and substituting into the utility function, maximization problem (4) becomes

\[
\max_x u = \max_x \left\{ \phi(x) + \psi \left( \frac{Y - (p_c x)}{p_c} \right) + \xi(T - t_i x) \right\}
\]

(5)

where the budgets have been internalized and the remaining decision variables involve the transportation commodities and \( p_c \). It is also possible to set \( p_c = 1 \) because all monetary variables can be measured relative to a general consumption price index.

Assuming that some travel is optimal \( (x > 0) \), the necessary and sufficient condition for maximization of (5) is then

\[
\phi'(x) - (p_c) \psi'(Y - p_c x) - t_i \xi'(T - t_i x) = 0
\]

(6)

where the "prime" symbols denote first derivatives of the functions.

In light of the requirements from economic theory that the utility components \( \phi, \psi \) and \( \xi \) each be monotonically increasing and quasi-concave, the following qualitative properties can be derived from solution condition (6). These properties reveal important fundamental characteristics of the general utility model. Proofs are given in Zinahvi (1979c).
(1) Travel (x) can never decrease as income (Y) increases.
(2) Travel can never decrease as available time (T) increases.
(3) Travel decreases with increasing costs. (In economic terms, the demand curve for travel
is always downward sloping.)
(4) Finally, travel increases with increasing speed.

Certain limitations of the model are evident from these derived properties. For instance, the
model does not allow the net substitution of activities which require less travel as income
increases; the net effect must be that travel increases with income, or at least remains the same.
And the same is true for increases in available time. Indeed, many of these simplifications are a
result of the short-term nature of the model: locations of residence and trip attractors are
assumed to remain fixed. Development of a complementary long-term residential location
model is underway (Zahavi et al., 1980a; Zahavi, 1980b).

The derived properties also show that the model is consistent with empirical evidence
regarding travel budgets and with the use of travel distance as a representation of the travel
quantity (x). In particular, (i) it has been noted that when travel speeds increase, travelers
prefer to trade-off saved time for longer trips, rather than for more trips; i.e. either for further
destinations, or for residence dispersion, rather than for more destinations (see Smith and
Schoener, 1978; Zahavi, 1979a). (ii) When incomes increase, travelers tend to purchase higher
speeds (such as by transferring from bus to car travel) and travel longer distances, instead of
generating more trips (Zahavi, 1979a). The use of trip rates alone to represent the quantity of
travel in the general utility model presents difficulties because utility should be independent of
the way in which trips are linked in the satisfaction of various travel purposes (Tanner, 1961;
Goodwin, 1979), except as these linkages affect travel times and costs.

In terms of distances traveled on various modes, i = 1, ..., m, and associated modal speeds
v_i and costs c_i, the general utility model of problem (5) can be restated as

$$\max_{x_i} u = \sum_{i=1}^{m} \phi_i(x_i) + \psi\left[ Y - \sum_{i=1}^{m} c_i x_i \right] + \xi \left( T - \sum_{i=1}^{m} x_i / v_i \right)$$

(7)

and the necessary and sufficient conditions for an optimum are

$$\phi_i(x_i) + \psi \left[ Y - \sum_{i=1}^{m} c_i x_i \right] - \frac{1}{v_i} \xi \left( T - \sum_{i=1}^{m} x_i / v_i \right) = 0, \quad i = 1, ..., m.$$  

(8)

Conditions (8) state that travel on each mode is adjusted to the point where the marginal benefit
gained is equal to the marginal cost incurred. The marginal cost is comprised of two terms, that
attributed to money and that attributed to time, where the marginal money costs are functions
of income and travel costs on all modes, and the marginal time costs are functions of available
time and speeds on all modes.

THE LOGARITHMIC UTILITY MODEL WITH ONE TRAVEL BUDGET

The implications of applying the utility model of expressions (7) and (8) to explain travel
decisions which might exhibit stable time and money budgets can be most clearly introduced by
first simplifying the model to the case of one budget, either money or time. The money budget
model applies to the minority of travelers to whom time is abundant relative to money, while
the time budget model applies to the minority of travelers to whom money is abundant relative to
time. The second step, pursued in the following section, is to analyze the more complicated
two budget form which might be more relevant to the majority of travelers. Comparisons of
results between the single-budget and two-budget forms reveal the simplifications of the former
and demonstrate the importance of considering time and money constraints simultaneously.

In light of the objectives of this development, the specification of functional forms for the \( \phi \),
\( \psi \) and \( \xi \) components of the general utility model is guided by the fact that only the natural
logarithm yields results which are consistent at first approximation with the existence of
constant travel budgets. That is, budget shares for any activity or subset of activities are
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constant if, and only if, the utility functions \( \phi, \psi \) and \( \xi \) are logarithmic. Specifically, the following theorems are restated from Zahavi (1979c), where they are developed from general postulates in mathematical economics (Beckmann and Künzi, 1979):

In the absence of an income budget, a constraint on available time for all activities implies a budget proportional to available time on total time spent for travel if, and only if,

\[
\mu = \sum_{i=1}^{n} a_i \log x_i + b_i \log \left( T - \sum_{i=1}^{n} x_i / y_i \right) + \text{const.} \tag{9}
\]

Analogously, in the absence of a constraint on time, an income constraint on total household expenditure implies a budget proportional to income on expenditure for travel if, and only if,

\[
u = \sum_{i=1}^{n} a_i \log x_i + b_i \log \left( Y - \sum_{i=1}^{n} x_i \right) + \text{const.} \tag{10}
\]

(For proofs see Zahavi, 1979c).

Focussing first on interpretations in money budget terms, that is, focussing on the case of travelers to whom time is abundant relative to money, the necessary and sufficient conditions for a maximization of eqn (10) are

\[
a_i / x_i = \frac{b_i c_i}{\left( Y - \sum_{i=1}^{n} x_i \right)} = 0, \quad i = 1, \ldots, m. \tag{11}
\]

Thus,

\[
c_i x_i = (a_i / b_i) \left( Y - \sum_{i=1}^{n} x_i \right), \quad i = 1, \ldots, m. \tag{12}
\]

which states that the total money expenditure on any mode is a constant proportion of income net of total travel expenses. The proportionality constant is the ratio of the attractiveness of the particular mode to the attractiveness of residual consumption, parameters which are to be estimated. Summing both sides of (12) over all modes,

\[
\sum_{i=1}^{n} c_i x_i = \left(1 / b_1 \right) \left( Y - \sum_{i=1}^{n} c_i x_i \right) \sum_{i=1}^{n} a_i. \tag{13}
\]

Since utility may be subjected to any monotone transformation, the \( a_i \) coefficients can be standardized such that \( \sum_{i=1}^{n} a_i = 1 \). Total travel expenditure then becomes

\[
\sum_{i=1}^{n} c_i x_i = \left( \frac{1}{b_1 + 1} \right) Y. \tag{14}
\]

Substituting (14) into (12), distance traveled on mode \( i \) is

\[
x_i = \frac{1}{c_i} \left( \frac{a_i}{b_i + 1} \right) Y, \quad i = 1, \ldots, m. \tag{15}
\]

\( ^{\dagger} \)Strictly speaking, the arguments of the logarithmic travel utility function are \( (x_i + 1) \), which establishes the proper boundary condition that utility is equal to zero at \( x_i = 0 \). Also, by expanding the arguments to \( (x_i + d_i) \), where \( \sum_{i=1}^{n} d_i = D \), a nondiscretionary amount of travel, \( D \), can be accounted for in addition to discretionary travel (Zahavi, 1979c). For simplicity, the added unit distance and compulsory travel allocated to each mode has been dropped in the present developments.
Thus, in the simplified single-budget model, total expenditure for travel is a fixed proportion of income, regardless of the costs of travel. Moreover, the total distance traveled on a given mode is inversely proportional to the cost per unit distance of travel on that mode.

Analogously, in terms of a travel time budget, the following two relationships are derived:

\[ \sum_{i=1}^{n} x_i/v_i = \left( \frac{1}{b_2 + 1} \right) T. \]  

(16)

and

\[ x_i = v_i \left( \frac{a_i}{b_2 + 1} \right) T, \quad i = 1, \ldots, m. \]

(17)

Thus, in the simplified single-budget model, total time spent traveling is a fixed proportion of total available time, regardless of travel speeds; and total distance traveled on any given mode is directly proportional to the speed on that mode.

Substituting the utility-maximizing travel distance, (15) and (17), into the original utility functions described in (10) and (11) respectively, yields the achieved utility levels. In the case of the time-budget model,

\[ u^* = \sum_{i=1}^{n} a_i \log \left[ v_i \left( \frac{a_i}{b_2 + 1} \right) T \right] + b_2 \log \left[ T - \left( \frac{1}{b_2 + 1} \right) T \right]. \]

(18)

Simplifying,

\[ u^* = \sum_{i=1}^{n} a_i \log a_i + \sum_{i=1}^{n} a_i \log v_i + (b_2 + 1) \log T \]

\[ - (b_2 + 1) \log (b_2 + 1) + b_2 \log b_2. \]

(19)

Thus, for the simplified single-budget model, eqn (19) shows that a household’s achieved utility, or total benefit from travel and leisure after travel is adjusted to yield maximum possible benefit, is an increasing concave function of travel speeds and of total available time. Also, achieved utility is a decreasing linear function of \(( - \sum a_i \log a_i )\), which is of an entropy form. In light of the well-known properties of such forms developed in information theory, achieved utility is greatest when modal attractions are highly unequally distributed, and is smallest when modal attractions are equal.

Analogous relationships are obtainable for the money-budget model which is relevant for the class of travelers to whom money matters little compared to time

\[ u^* = \sum_{i=1}^{n} a_i \log a_i - \sum_{i=1}^{n} a_i \log c_i + (b_1 + 1) \log Y + \text{const.} \]

(20)

A household’s achieved utility is a decreasing convex function of costs, an increasing concave function of income and a decreasing linear function of the entropy of modal attractions.

Equations (19) and (20) can be interpreted with a view to policy implications. Focusing on eqn (19): achieved utility increases with time available and with any modal speed in a diminishing marginal manner. Moreover, increases in speeds faced by households with low levels of available time (say, more workers per total travelers) have greater proportional effects on achieved utility than do increases in speeds faced by households with high levels of available time (tastes being equal). Finally, the incremental increase in a household’s achieved utility resulting from an increase in the speed on any mode is proportional to the attraction of the mode and inversely proportional to the present average speed of the mode. Assuming that faster modes are also more attractive for reasons other than travel times alone (say, for reasons related to comfort or personal security), the two effects of high speed and high attraction counteract, and it is possible that achieved utility can be improved to the same extent by
improvements in speeds of the either faster or slower mode. Results depend upon the actual estimated values of the $a_i$ and $v_i$ terms. Similar interpretations of achieved utility as a function of time availabilities and travel speeds have been reported by Burns (1979), using concepts of time-geography (Hägerstrand, 1970 and 1974; Lenntorp, 1970 and 1976). This correspondence of results is encouraging, but comparisons with results from the two-budget model developed in the following section highlight the simplifications inherent in the time-geography approaches.

**THE LOGARITHMIC UTILITY MODEL WITH TWO TRAVEL BUDGETS**

In the simultaneous presence of time and money constraints, these constraints interact, and travel time and money expenditures are not simply constant as travel times and/or costs vary. In economic terms, flexible rather than rigid expenditure patterns are to be expected. In the two-constraint case, the question concerning the usefulness of travel budget information in travel demand forecasting then translates into a question of the appropriateness of linear approximations to nonlinear phenomena. The present section explores the conceptual issues involved in this approximation question, while later sections present some preliminary empirical evidence.

The complete two-constraint model is specified as follows:

$$\max u = \sum_{i} a_i \log x_i + b_1 \log \left( Y - \sum_{i} c_i x_i \right) + b_2 \log \left( T - \sum_{i} v_i x_i \right)$$  \hspace{0.5cm} (21)

and the necessary and sufficient conditions for maximization are

$$\frac{a_i x_i}{b_1 c_i} = \frac{b_2}{v_i \left( T - \sum_{i} v_i x_i \right)} \hspace{0.5cm} i = 1, \ldots, m.$$  \hspace{0.5cm} (22)

In general, conditions (22) represent an intractable set of non-linear equations. Gradient-search and similar algorithms (Bard, 1974) can be used to find approximate solutions in practical applications. However, certain approximations can be introduced in order to develop closed-form solution. These solutions reveal the implications of employing the model in travel demand forecasting and also reveal the simplifications imposed in time-allocation studies where money constraints are ignored.

One approximation involves assuming that total travel expenditure is a relatively small proportion of income, and that total time spent traveling is a relatively small proportion of time available. A conventional definition of "small" in such cases is typically "ten per cent or less", but 11 or even 15% might be acceptable. Defining

$$\sum_{i} c_i x_i \ll Y$$  \hspace{0.5cm} (23)

and

$$\sum_{i} x_i / v_i \ll T,$$  \hspace{0.5cm} (24)

condition (22) becomes:

$$x_i = \frac{a_i}{\left( b_1 c_i / Y \right) + \left( b_2 v_i / T \right)} \hspace{0.5cm} i = 1, \ldots, m.$$  \hspace{0.5cm} (25)

Thus, assuming that travel time and money budgets are relatively small proportions of total money and time, travel on any mode is proportional to the harmonic mean of two distance limits: (1) $Y/c_i$, the maximum distance that can be reached with the available money, and (2) $v_i T$, the maximum distance that can be reached with the available time.

Zahavi (1979c) applies expression (25), and similar expressions, as foundation for the travel choice equations within the UMOT Model.
Comparing equations (25) with the optimum travel distance equations for the two alternative single-budget models (15) and (17), the simultaneous presence of two budgets introduces nonlinearities between travel distances and income or the inverse of costs, and between travel distances and available time or speeds. These relationships become concave functions. Income, the inverse of costs, available time, and speeds all exhibit diminishing marginal influences on travel distance. Thus, for example, a certain change in income will have a greater effect on travel for households at lower income levels than it will for households at higher income levels. *ceteris paribus.*

The simultaneous presence of time and money constraints thus leads to deviations from travel money and time budgets which are constant proportions of income and time available, respectively. The degree to which a linear assumption of constant expenditure proportions fits this theoretically nonlinear situation is an empirical research question. However, it is quite possible that such nonlinearities might be approximated in practice by stratifying households by income level and available time (number of travelers, number of workers and possibly type of work) and specifying different linear relationships for each stratum. Similarly, different cost-distance or speed-distance relationships could be specified for various ranges of costs or speeds, as is proposed in the UMOT Model of Zahavi (1979c).

One further implication of the two-budget model is the resulting expression for the ratio of the marginal utility of time to the marginal utility of money:

$$\frac{du}{dT} \div \frac{du}{dy} = \frac{b_1}{b_2} \frac{Y - \sum c_k x_k}{(T - \sum x_k v_k)}$$

(26)

Thus, the value of time implied by the two-budget logarithmic utility model is directly proportional to money available for non-travel consumption and inversely proportional to time available for non-travel discretionary purposes. The proportionality constant reflects the taste of the household, or segment of households for which the model is calibrated. Furthermore, the value of time given by expression (26) is a function of travel conditions reflected in travel costs $c_k$ and travel speeds $v_k$. In general these conditions might be dependent on the time of day (say, peak versus non-peak) as well as on the transportation system characteristics of the urban area in question or the household's location within an urban area. Similarly, taste coefficients ($b_1$ and $b_2$) might differ by household segment as defined by sociodemographic and life style variables. The sum total of all these effects is a realistically variable value of time concept which takes into account both supply-side and demand-side variables and requires careful data analysis.

It is shown in Zahavi (1979c) that the present utility theory approach can be extended to account for car ownership decisions, as well as the determination of travel distance by mode. Thus, the UMOT Model of Zahavi (1979c) employs linked travel and car ownership choice models which are based on consistent theoretical hypotheses. Description of the car ownership model is beyond the scope of the present paper.

**Empirical Examples**

The following empirical examples (Zahavi, 1979c) are presented for illustrative purposes only and are not purported to be adequate tests of the theory. Adequate tests await applications to disaggregate data processed especially for the research questions at hand. The present applications use available aggregate data, requiring only limited data processing and statistical calculations.

Beginning with a simplest case, it is shown in Zahavi (1979c) that when the number of modes equals two, the two budget equations of the utility model alone determine the optimal $x_1, x_2$ solution to utility maximization. Applying this simplified version of the model to aggregate, district-level travel data for Washington, D.C. in 1968 (Zahavi, 1979c) results in the estimated daily travel distance by mode per representative household are detailed in Table I. Figure 1 shows the estimated travel distance per household, by mode, as continuous curves, and the observed values as dots. The correspondence between the estimated and the observed values is encouraging, especially in light of the fact that the estimated values were not calibrated to
observed proportions of travelers using the two modes, but were derived from the observed travel budgets and theoretical utility relationships.

The data in Table 1 can also be expressed in a different way, as shown in Fig. 2. The diagram details the daily travel distance per household that can be generated by each mode within each travel budget separately (i.e., by dividing each budget by the unit costs of each mode). Since the faster mode usually is the more expensive mode, the travel distance that can be realized within the two constraining budgets simultaneously is expected to be within the shaded area in Fig. 2, within which lies the maximum travel distance that can be generated by using combinations of the available modes, as detailed in Table 1.

The relationships in Fig. 2 also suggest what possible shifts in modal choices are to be expected if travel conditions change. For example, increasing the unit cost of car travel will lower the car-TM curve, thus resulting in (i) a wider choice-set, (ii) an increase in bus travel, and (iii) a decrease in total daily travel distance. The last result is of considerable importance, as it suggests that modal transfers are not one-to-one transfers (as usually is the case when mode choice is based on trips) since travel may be gained or lost, depending on the direction of transfer.

The figure further suggests that representative households with an annual 1968 income within the range of $4000-$15,500 can utilize both travel budgets in trade-offs to achieve maximum travel benefits. There are cases, however, where one budget alone is binding. For

<table>
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<th>Annual Income, $</th>
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<th>5,000</th>
<th>6,000</th>
<th>7,000</th>
<th>8,000</th>
<th>9,000</th>
<th>10,000</th>
<th>11,000</th>
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<td>0.72</td>
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<tr>
<td>Money Budget, $</td>
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<td>1.12</td>
<td>1.20</td>
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<td>1.37</td>
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<td>Car Travel:</td>
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<td></td>
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<tr>
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<td>2.14</td>
<td>2.14</td>
</tr>
<tr>
<td>Unit cost, $/km.</td>
<td>0.096</td>
<td>0.062</td>
<td>0.084</td>
<td>0.16</td>
<td>0.24</td>
<td>0.34</td>
<td>0.42</td>
<td>0.56</td>
</tr>
<tr>
<td>Distance, km.</td>
<td>39.4</td>
<td>30.9</td>
<td>23.4</td>
<td>17.7</td>
<td>13.6</td>
<td>10.8</td>
<td>9.6</td>
<td>7.4</td>
</tr>
<tr>
<td>Bus Travel:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit time, min/km.</td>
<td>8.82</td>
<td>7.40</td>
<td>6.32</td>
<td>5.71</td>
<td>5.00</td>
<td>4.62</td>
<td>4.29</td>
<td>4.07</td>
</tr>
<tr>
<td>Unit cost, $/km.</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
<td>0.037</td>
</tr>
<tr>
<td>Distance, km.</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
<td>13.65</td>
</tr>
<tr>
<td>Total Distance,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>km./Household</td>
<td>13.65</td>
<td>16.35</td>
<td>20.92</td>
<td>30.75</td>
<td>41.11</td>
<td>50.25</td>
<td>67.15</td>
<td>67.15</td>
</tr>
</tbody>
</table>

Table 1. Summary of estimated travel distance per household, by income, averaged by district, Washington, D.C., 1968

Fig. 1. Estimated vs observed daily travel distance per household, by mode vs income by district, Washington, D.C., 1968.
instance, representative households below an annual 1968 income of approx. $4000 are constrained in their travel choices by money expenditures alone, while households above an annual 1968 income of approx. $15,500 are constrained by time expenditures alone. Such cases analytically operationalize the planning concepts of "captive" riders on particular modes. Moreover, these cases are of special interest since they indicate segments which are potential target markets for new or altered modes.

The second sample application of the UMOT Model travel choice equations involves data from the Nuremberg Region in the Federal Republic of Germany (Zahavi, 1979c). Estimates of travel distance by mode and maximum daily travel distance generated within each travel budget were calculated in the same way as for Washington, D.C. However, since no direct income data were available, households were stratified by size and car ownership. Results are shown in Tables 2 and 3 and Figs. 3 and 4. It is evident once again that the mode-choice set is relatively narrow, as shown by the shaded area in Fig. 4, and that the observed choices tend to be located at the center of this choice set.

**HOUSEHOLD TRAVEL BUDGET DETERMINATION**

An important dynamic issue is how travel time and travel money budgets, $T^*$ and $M^*$, are determined over the long run by a household. That is, while travel budgets might be considered

<table>
<thead>
<tr>
<th>Household Size</th>
<th>Cars per Household</th>
<th>Daily Distance, km</th>
<th>Travel Modal Split, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3.45, 12.31</td>
<td>32.94, 78.1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>6.87, 19.55</td>
<td>82.92</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>39.79, 51.7</td>
<td>45.55, 12.7</td>
</tr>
<tr>
<td></td>
<td>2+</td>
<td>67.63, 0.90</td>
<td>68.73, 11.2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>8.28, 24.01</td>
<td>33.99, 75.0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>37.76, 13.65</td>
<td>52.49, 26.3</td>
</tr>
<tr>
<td></td>
<td>2+</td>
<td>67.97, 6.70</td>
<td>73.77, 9.1</td>
</tr>
<tr>
<td>4+</td>
<td>0</td>
<td>14.03, 36.28</td>
<td>50.31, 72.1</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>42.61, 25.39</td>
<td>60.60, 37.3</td>
</tr>
<tr>
<td></td>
<td>2+</td>
<td>80.21, 18.21</td>
<td>98.42, 18.5</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>32.81, 12.96</td>
<td>45.56, 28.1</td>
</tr>
</tbody>
</table>
Table 3. Maximum daily travel distance per household that can be generated by mode within each travel budget:

<table>
<thead>
<tr>
<th>HH Size</th>
<th>Cars/HH</th>
<th>Money Budget</th>
<th>Time Budget</th>
<th>Unit Cost</th>
<th>Unit Time</th>
<th>Distance F</th>
<th>Distance T</th>
<th>Distance O</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.37</td>
<td>6.32</td>
<td>1.30</td>
<td>4.33</td>
<td>1.10</td>
<td>2.64</td>
<td>5.99</td>
<td>18.5</td>
</tr>
<tr>
<td>2</td>
<td>0.82</td>
<td>12.36</td>
<td>2.61</td>
<td>3.33</td>
<td>1.11</td>
<td>2.33</td>
<td>4.99</td>
<td>36.9</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>15.35</td>
<td>2.99</td>
<td>3.52</td>
<td>1.18</td>
<td>2.36</td>
<td>4.65</td>
<td>43.6</td>
</tr>
<tr>
<td>4+</td>
<td>1.21</td>
<td>17.38</td>
<td>3.64</td>
<td>3.32</td>
<td>1.22</td>
<td>2.25</td>
<td>4.48</td>
<td>59.1</td>
</tr>
</tbody>
</table>

HH - Household
Money Budget - in DM
Time Budget - in Hours
Unit Cost - DM/km.
Unit Time - Min./km.
P.T. - Public Transport

Fig. 3. Modal split by distance, by household size and car ownership, the Nuremberg region, 1975.

Fig. 4. Maximum daily travel distance per household under the travel time and money budgets vs car ownership levels, the Nuremberg region, 1975.
fixed (or approximately fixed for an household segment) in the short run, these budgets can be expected to change in the long run as household incomes or available times change. In economic terms, expenditures are fixed in the short run but become flexible in the long run when utility maximization is applied to the expenditures themselves, and not just to their allocation. Budgets in one model are variables in a higher-level model.

The decision problem faced by an household over the long run can be specified in utility terms as:

$$\max_{T^*, M^*} u = a \log \left( \sum x_i \right) + b_1 \log (Y - M^*) + b_2 \log (T - T^*)$$  \hspace{1cm} (27)

where the fixed travel money budget is given by

$$M^* = \sum_{i=1}^n c_i x_i$$  \hspace{1cm} (28)

the fixed travel time budget is given by

$$T^* = \sum_{i=1}^n x_i / v_i$$  \hspace{1cm} (29)

and, for simplification, the argument of the travel utility function is total travel. Here it is proposed that the household is faced with choices involving trading off the utility from total travel, $\phi(\sum x_i)$, against consumption and leisure utilities in determining travel budgets at this long-run stage of the travel decision process. The necessary and sufficient conditions for an optimum in (27) are then:

$$\frac{a}{\sum x_i} - \frac{b_1 c_i}{(Y - M^*)} - \frac{b_2}{v_i (T - T^*)} = 0, \hspace{1cm} i = 1, \ldots, m.$$  \hspace{1cm} (30)

Conditions (30) are not descriptive of households' allocation among modes; that is the purpose of the short run model presented in the previous sections. Rather, the implications of conditions (30) are revealed by writing the conditions for any two modes, say $i=1,2$, and equating:

$$\frac{b_1 c_1}{(Y - M^*)} + \frac{b_2}{v_1 (T - T^*)} = \frac{b_1 c_2}{(Y - M^*)} + \frac{b_2}{v_2 (T - T^*)}$$  \hspace{1cm} (31)

Thus,

$$T - T^* = (Y - M^*) \left( b_2 \left( \frac{1}{v_1} - \frac{1}{v_2} \right) \right) / (b_1 (c_1 - c_2))$$  \hspace{1cm} (32)

The coefficient in eqn (32),

$$\beta = \left( b_2 \left( \frac{1}{v_1} - \frac{1}{v_2} \right) \right) / (b_1 (c_1 - c_2))$$  \hspace{1cm} (33)

relates time for leisure to money for residual consumption. It is a function of the sum of modal costs and the sum of the inverse of modal speeds. It is positive if one mode does not dominate the other (that is, if $c_1 > c_2$ for $v_1 > v_2$, or $c_1 < c_2$ for $v_1 < v_2$), a condition expected to be satisfied in the case of car (faster, more expensive) versus public transit (slower, less expensive). Substituting definition (33) in eqn (32) and solving for the travel time budget,

$$T^* = (T - \beta Y) + \beta M^*.$$  \hspace{1cm} (34)
That is, travel time and money budgets are linearly related over the long-run. This is testable by observing the time and cost expenditures on travel by the same or similar households over time as incomes or available times change. The slope of the derived linear relationships is independent of income and available time, and dependent on travel costs and speeds. The intercept is dependent upon income and available time as well as travel times and costs and is positive for $T > \beta Y$; that is, it is positive in cases where time is relatively more abundant in relation to money.

Consistent with usual travel demand model estimation procedures, the above results is also testable to a certain extent using data for a cross-section of households at one point in time. Figure 5 shows two examples of the relationship between households' travel time and money budgets for three car ownership segments in the Nuremberg region and the metropolitan area of Munich (Zahavi, 1979c). The parameters of the linear regressions plotted in Fig. 5 are listed in Table 4. Table 5 lists results of hypothesis tests (Chow, 1960) that the parameters of the regressions for each car ownership segments are equivalent for Nuremberg and Munich. The results indicate the following:

1. For each car ownership segment in each area, there is a linear relationship between time and money budgets, as predicted by eqn (34).
2. For both the areas, there are significant differences among the car ownership segments in terms of both the intercepts and slopes of the linear relationships, indicating that car ownership is an effective criterion by which to segment households into homogeneous segments with respect to relationships between allocated time and money travel budgets.
3. The major difference in intercept values among the car ownership segments is consistent with the role of car ownership as an income stratification (the term $(T - \beta Y)$ in eqn (34) decreases with increasing numbers of cars owned).
4. The time-money budget relationships for zero-car households are not significantly different, Nuremberg vs Munich. But the relationships are different between the two areas for

![Fig. 5. Daily travel time and money expenditures per household, Nuremberg and Munich.](image-url)
Table 4. The relationships between travel time and money budgets per household, Nuremberg and Munich

<table>
<thead>
<tr>
<th>Regression</th>
<th>NUREMBERG</th>
<th>MUNICH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>0 Car</td>
<td>1 Car</td>
</tr>
<tr>
<td>Slope</td>
<td>0.559</td>
<td>0.469</td>
</tr>
<tr>
<td>Intercept</td>
<td>6.598</td>
<td>-4.914</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.958</td>
<td>0.925</td>
</tr>
</tbody>
</table>

Table 5. Comparison of regressions, Nuremberg vs Munich

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>F - Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>0 Car</td>
</tr>
<tr>
<td>Equality of</td>
<td></td>
</tr>
<tr>
<td>Regressions</td>
<td>1.58</td>
</tr>
<tr>
<td>(2, 26)</td>
<td>(2, 51)</td>
</tr>
</tbody>
</table>

(-1, -1) Degrees of Freedom

1-car and 2+ car households. This indicates that one or both of the following might be true: average speeds are lower for Munich; or average costs are higher for Munich. Both possibilities are consistent with the fact that the data for Nuremberg are for an extended region, while the data for Munich are for a higher density urban area.

CONCLUSIONS

Investigations into travel time and money budgets have been primarily empirical in focus. Using concepts and mathematical functions proven to be effective in modeling consumer behavior in microeconomics, the present approach attempts to establish theoretical hypotheses for testing the predictability of budgets and their potential role in travel demand forecasting. Results are encouraging. In the short run, household allocations of scarce time and money resources among travel alternatives can lead under certain conditions to constant expenditure proportions; under other conditions such constant proportions are only approximations to nonlinear phenomena. In the long run, a model of household determination of budget levels in light of available resources and money and time costs is shown to be tractable and consistent with empirical evidence. There are two obvious directions for further research: extension of the theory and resolution with alternative proposals, and empirical testing with more detailed and encompassing travel data.

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REFERENCES

A utility-theory travel demand model incorporating travel budgets


