THE EFFECTS OF TRANSPORTATION SYSTEMS ON THE
SPATIAL DISTRIBUTIONS OF POPULATION AND JOBS

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ABSTRACT

Most urban structure models are based on some form of an assumed formula that tries to express the spatial location of population by the job location, the transportation system, and other similar factors. There are several difficulties with this approach, one of which is the problem of transferability, when each model has first to be fully calibrated to the local conditions of each city before it can be regarded as valid and suitable for application.

This paper presents the results of an empirical analysis of a wide range of cities, from different countries, indicating that:

1. The differential accumulation of population vs. Job locations, expressing the difference of the average distances of the two distributions from the city's center, approximates the average trip distance in the city;

2. A Gamma function can describe the differential accumulation, where the ratio between the alpha and beta parameters approximates the average trip distance;

3. All cities can then be classified and ranked by their two parameters, expressing their internal structure by the interaction between the population and the job spatial distributions;

4. When travel is constrained by the households' daily money and time budgets, increases in travel speeds do not seem to result in saved travel times, as such, but in increased travel distances and greater spatial opportunities. It is then shown how increases in travel speeds, brought by improving the transportation systems, can change mobilities and urban structure.

The proposed model appears to be fully transferable between cities. It also combines travel demand, system supply and urban structure by feedback loops under a Unified Mechanism of Travel (UMOT) within the two principal travel budgets, of money and of time. The UMOT model is being developed for the World Bank, Urban Projects Department, as a macro tool for rapid assessments of future transportation systems and urban structure alternatives.
* Urban structure is a complex mechanism, representing a compromise between a variety of interacting factors - geographic, historical, social and economic, to name just a few. Hence, it is practically impossible to describe all the nuances of urban interactions, and the resulting urban structure, by one process and still keep the process simple and controllable (1,2,3). Thus, this paper discusses only one aspect of the complex urban system, namely the effects of the transportation system supply on urban structure, and although it appears to be a major aspect, it should be regarded within its limitations.

1. THE UMOT PROCESS

1.1 The first part of this paper describes in a telegraphic way a Unified Mechanism of Travel, UMOT for short. The UMOT process has been developed for the World Bank, Urban Projects Department, as a macro tool for rapid assessment of alternative transportation and urban plans, although the process itself is basically disaggregate in concept and application (4). The main subject of this paper - urban structure - will be elaborated only in the second part of the paper, since it is only one component of the overall UMOT process.

It should be noted at this stage that since the UMOT has been developed on a strictly empirical basis, mainly on the transportation studies of Washington, D.C. (1955 & 1968) and Twin Cities (1958 & 1970), with the corroborating of additional data from other studies (5), the results are presented as indicators only, that have still to be verified beyond any doubt. Nonetheless, the emerging understanding of the mechanism of travel appears to be consistent not only with established relationships in other fields, such as economics and urban geography, but it also seems to unify the interactions between travel demand, system supply and urban structure under a consistent behavioral mechanism.

1.2 The foremost requirements for a transportation/urban model may be defined by the following characteristics, part of which are overlapping:

(1) It should explicitly express cause and effect, based on a behavioral mechanism. For instance, instead of using observed travel characteristics for the calibration of a model, the model should rather produce the observed travel characteristics as outputs when based on the minimum number of descriptors of the population, the system supply, and the urban structure;

(2) The model should be fully transferable between cities, with no need for recalibration. Put in another way, its ability to forecast travel characteristics to a future date for one city
should be verified by its ability to estimate travel characteristics at base year conditions for other cities;

(3) The model should explicitly describe and explain the interaction between travel demand, system supply and urban structure by feedback loops between all three. For instance, the model should be able to explain the change of the trip rate with city size, or the expansion of a city with increased travel speeds, as a natural outcome from such an interaction, and not as isolated and independent phenomena.

(4) Last, but not least, the same model should also be responsive to a wide range of policy issues, from changes in the components of travel costs (e.g., gas, transit fares, road pricing) to changes in the levels of land use control, and their effects on the amount and patterns of travel, as well as on the expected changes in urban form and structure.

As can be seen from the above, it is a tall order and, with all frankness, the UMOT at this stage is still a process and not such a model. It does, however, show several promising signs and potentiality for being developed into such a model, foremost of which is its ability to unify the interaction between travel demand, system supply and urban structure under a behavioral mechanism.

1.3 The UMOT process may be summarized by the following indicators:

(1) Households are willing to allocate a certain proportion of their income to travel, a proportion that is relatively stable both between cities and over time;

(2) Trip-makers tend to have a daily door-to-door travel time budget, which is stable both between cities and over time;

(3) Trip-makers strive to maximize their daily travel distance within the constraints of their household's money budget and their own time budget, thus striving to maximize their spatial opportunities;

(4) The fundamental travel relationship, in a simplified form, is:

$$\frac{M}{T} = \frac{sc}{v}$$

where the household's money budget over its time budget equals the product of the average speed of travel and the average cost per unit distance travelled at that speed;

(5) While the left hand side of the equation represents travel demand in terms of cost and time budgets, the right hand side represents the product that the household would like to purchase from the transportation system supply, in terms of the system's performance and the price of using it;

(6) Households at increasing income levels wish, therefore, to purchase travel at increasing speeds if equilibrium, and satisfaction, are to exist between travel demand and system supply;
(7) Households owning a car tend to allocate a stable proportion of their income to travel, at about 11-13 percent, at all income levels. However, households not owning a car expend only 3 to 5 percent of their income on travel, again at all income levels. (The proportions are somewhat higher when related to total expenditures, instead of income, although their stability remains unaffected);

(8) Hence, households owning a car are found to have better opportuni-ties for reaching equilibrium conditions between their travel demand and system supply. Conversely, households not owning a car, that have to travel by the slower transit, expend their travel time budget much before they reach even half of their travel money budget. Hence the strong incentive to purchase a car with increase in income;

(9) It is thus indicated that travel demand (in the general sense, including latent demand) may not necessarily be in equilibrium condition with system supply when applied to urban structure. Put in another way, it may well happen that we should rather seek the amount of disequilibrium in the urban system, as representing a behavioral force vector that changes and reshapes urban structure, rather than concentrate on the concept - and techniques - of equilibrium conditions;

(10) The households' efforts to reach equilibrium conditions between their travel demand and system supply seem to result in shifts and changes in the spatial distributions of residence and jobs. The search for higher speeds and wider spatial opportunities tends to disperse households with higher incomes outwards from the congested center to areas where higher speeds can be achieved. Furthermore, if system improvements increase the speed of travel relatively more around the city rather than within it (such as by a beltway), the dispersion of residences will be encouraged and accelerated;

(11) Tripmakers tend to trade off increases in speed for longer trips than for more trips, within their stable daily travel time budget. The cross elasticities between the two trade offs depend on the gradient of population density with distance from the center, thus giving more accent to longer trips and dispersion when the base year gradient is steep, and less accent when the base year gradient is gentle;

(12) If, however, speeds start to decrease because of congestion, tripmakers have the choice of decreasing their trip distance, their trip rate, or a combination of both. Since their perceived value of the trip rate tends to follow a decreasing marginal utility relationship, their reluctance to decrease the trip rate grows rapidly at low rates. In such cases, tripmakers will have to shorten their trip distances, by either relocating their residence back towards the city center, a choice which most people tend to reject, or relocating their jobs nearer to their residence, which appears to be the preferred choice. The end result is that jobs start to follow residences outwards in order to shorten trip distances, thus changing mono-nucleated cities into multi-nucleated ones;
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(13) In the cases where cities are allowed to expand freely, the above process is well known, and feared, since it starts with population dispersion and income polarization, and ends up with deterioration of the cities' centers, which are left by both the high income classes and the jobs that are closely related to and linked with them;

(14) Transit is at the losing end of modal splits, since its operating speed is always far below the speed of cars (about 50 percent only). Hence, it is not the transit fare which is the important factor for inducing car tripmakers to transfer to transit, but the speed of travel. This is why rapid transit (either rail or bus) becomes so important at certain moments in the life of large cities. (Although this subject is beyond the scope of this paper, it should be noted that rapid transit has its own dangers to the life of cities since if not planned rightly, it may enhance the drainage of the city center, and transfer land uses and activities to the radial corridors);

(15) Another point of interest is the indication that tripmakers regard transportation as a superior good, in the economic sense, thus they are willing to expend more money on better (i.e., faster) transportation in order to travel longer distances and increase their spatial opportunities, with the expectation of increasing their benefits including their incomes, rather than save travel time and money and travel less. It might, therefore, come as a surprise that an increase in speed (within the speed range found in cities) increases the expenditures on travel, while congestion decreases the expenditures on travel, within the same travel time budget. Hence, the danger of congestion is not so much the loss of travel money and travel time to the tripmakers as the loss of mobility and opportunities, in both the economic and the social implications. Put in another way, a decrease in the speed of travel will not only tend to slow down increases in income on the one hand, but it will also tend to increase the disequilibrium between travel demand and system supply, thus increasing dissatisfaction and frustration of the population on the other hand;

(16) The above mechanism also explains the ranking of varying proportions of trip purposes under different travel conditions. For instance, the more congested the peak period is, the less travel is observed in the evening, since just to travel to work and back has taken up most, or all of the travel time budget, thus leaving no travel time for other purposes, such as social-recreation. Hence, the number of trips and the proportions of their purposes, as well as the spread of peak hour into peak period travel, are finale outputs from the UMOT process.

1.4 The foregoing constitute a brief - and it is brief for such a complex subject - discussion of the UMOT process, indicators, and their implications. The graphic presentation of the UMOT process, for car travel, is shown in Figure 1, where the interactions between travel demand, system supply and urban structure are characterized by feed back loops. The emphasis will now be shifted to the specific subject of urban structure and the simultaneous interaction between the spatial distributions of population and jobs in the urban area.
Figure 1. Schematic Flow Chart of the Unified Urban-Transport Model (UMOT) (Private Transport)

A. Population

Population Characteristics
- Income
- Motorization
- Tripmakers
- Travel Budgets
  - Cost
  - Time
  - Unit Cost
  - Daily Travel Distance
  - Equilibration
  - Trip Rate
  - Trip Purposes
  - Perceived Value of Travel
  - Trip Frequencies

B. Transport System

System Supply
- Exp.
- Art.
- Loc.
- Private Vehicles
- C.V. + Other
- The Alpha Relationship
- Speed
- VMT
- Trip Distance

C. Urban Structure

Urban Structure
- Physical Limitations on Expansion of Area
  - Population Spatial Distribution
  - Jobs Spatial Distribution
  - Differential Accumulation
  - Rent Distribution

Short Range Iterations

Long Range Iterations

--- Inputs sensitive to alternative policies
--- Outputs
2. **THE DIFFERENTIAL ACCUMULATION PROCESS**

2.1 Most analyses of population and job spatial distributions regard and treat each distribution separately, and allocate one to the other on the basis of some assumed interaction mechanism, such as the gravity and the intervening opportunities models, or their derivatives. Even when the allocation process is a simultaneous one, each distribution still retains its identity in the process. The problem with such a methodology is that it requires not only the treatment of two separate spatial distributions, but also the calibration of the assumed interaction mechanism between the two to observations.

The problem becomes rapidly more involved when each distribution is further stratified by classes of population, land uses and activities, where each class has to be calibrated separately. Since the discussion of such and similar problems are available in many reviews and summaries (c,2), this paper will mention only one aspect of such models, namely their heavy reliance on transportation characteristics as measures for the spatial separation and interaction between the urban activities, culminating in the spatial allocation of population and jobs in the urban area.

2.2 Generally, allocation models are based on two important outputs from transportation studies: the number of trips, by purpose, generated and terminated in the zones, and a measure of the spatial separation between the zones, expressed in either absolute units such as time and money (transformed into 'generalized costs') or in relative units such as attractiveness and accessibility indices.

It should be noted that in most transportation models, trip generation is the first phase, mostly dependent on socio-economic characteristics and seldom - and then implicit only - dependent on the transportation system and urban structure. The UMOT process, on the other hand, indicates that trip generation should be considered as the last, rather than the first, phase in the transportation process. For instance, the size of a city appears to have an important effect on the trip rate, as can be seen in Table 1 and Figure 2. Hence, alternative predictions of the future size of the city may have different effects on the amount and patterns of trips, even for the same socio-economic characteristics. For example, while cross-sectional analyses suggest that the trip rate increases with income, motorization, accessibility, and the like, the trip rate may actually decrease if the city will double or triple in size, even if all the cross-sectional explanatory variables will increase with time. Thus, if trips, which seem to be affected by city size (in the general sense, including their number and spatial distribution within the city) are used to forecast city structure, a recursive bias may be introduced.

2.3 With these, and similar problems in mind, a search was initiated for a direct description of the spatial distributions of population and jobs, and their responsiveness to and interaction with travel behavior and system supply. The process, as derived from empirical analysis of data from a wide selection of cities as well as over
Figure 2. Car Daily Trip Rate vs. City Size

Table 1. Population Size and Travel Characteristics in a Selection of Cities

<table>
<thead>
<tr>
<th>No.</th>
<th>City</th>
<th>Year</th>
<th>Population</th>
<th>Car Daily Trip Rate</th>
<th>Trip Time min.</th>
<th>Trip Distance miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Monroe</td>
<td>1965</td>
<td>96,530</td>
<td>5.79</td>
<td>7.3</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>Baton Rouge</td>
<td>1965</td>
<td>245,076</td>
<td>4.66</td>
<td>n.a.</td>
<td>3.3</td>
</tr>
<tr>
<td>3</td>
<td>Peoria</td>
<td>1964</td>
<td>260,826</td>
<td>5.10</td>
<td>n.a.</td>
<td>4.1</td>
</tr>
<tr>
<td>4</td>
<td>Orlando</td>
<td>1965</td>
<td>355,619</td>
<td>4.33</td>
<td>9.7</td>
<td>4.3</td>
</tr>
<tr>
<td>5</td>
<td>Springfield</td>
<td>1965</td>
<td>532,188</td>
<td>4.30</td>
<td>n.a.</td>
<td>4.9</td>
</tr>
<tr>
<td>6</td>
<td>Cincinnati</td>
<td>1965</td>
<td>1,391,869</td>
<td>3.63</td>
<td>13.7</td>
<td>5.5</td>
</tr>
<tr>
<td>7</td>
<td>Baltimore</td>
<td>1962</td>
<td>1,607,980</td>
<td>3.26</td>
<td>12.3</td>
<td>5.8</td>
</tr>
<tr>
<td>8</td>
<td>Washington, DC</td>
<td>1968</td>
<td>2,562,025</td>
<td>3.28</td>
<td>15.6</td>
<td>6.6</td>
</tr>
<tr>
<td>9</td>
<td>Los Angeles</td>
<td>1960</td>
<td>7,595,834</td>
<td>3.66</td>
<td>13.1</td>
<td>8.2</td>
</tr>
<tr>
<td>10</td>
<td>Tri-State</td>
<td>1964</td>
<td>16,303,000</td>
<td>2.89</td>
<td>20.1</td>
<td>9.9</td>
</tr>
</tbody>
</table>
time, was called the Differential Accumulation process, as detailed below. It should be noted that the following description is kept to a macro level, in broad-brush terms, in order to illuminate the basic principles. Further elaboration would be done at a later stage.

2.4 The location of workers in a city is described by the distribution of their distances from the city center. (If worker locations are not available, then a surrogate may be used, such as households or population locations, for approximative estimation). Similarly, the job locations are also described by the distribution of their distances from the city center. The differential accumulation process is now defined in the following way:

Let $0 < x_1 < x_2 < \ldots < x_n$ be the mid-point distances of rings about the center of the city. Let $p(x_i)$ and $e(x_i)$ be the fractions of workers and jobs, respectively, in the ring associated with $x_i$.

Define $g(x_i)$ by:

$$g(x_n) = p(x_n) - e(x_n) ;$$
$$g(x_{n-1}) = p(x_{n-1}) + p(x_n) - e(x_{n-1}) - e(x_n) ;$$
$$\vdots$$
$$g(x_k) = \sum_{j=k}^{n} [p(x_j) - e(x_j)] ;$$
$$g(x_1) = 0 ;$$

(2.1)

The function $g(x_i)$ is called the differential accumulation function since it represents the accumulated differences between the two distributions, of workers and jobs. Examples of such functions, normalized by percent (i.e., the total number of workers or their surrogate, and the total number of jobs equal each 100 percent) are shown in Figure 3 for a selection of cities in both developed and developing countries.

2.5 It becomes apparent from Figure 3 that the differential accumulation curves (henceforth d/a, for short) are similar in form, although they vary in scale and shape; they increase slowly towards the center, reaching a peak near the center and then fall sharply down to zero at the center itself. The few cases where a second bump appears near the edge of the city represents either regional characteristics or an expressway around the city, as discussed later.

Since in all cities analyzed the number of workers was greater than the number of jobs when starting from the edge towards the city center, the curves represent the proportion of workers who have to travel towards the city center in order to reach working places. Hence, the scale and shape of the curves should represent some measures of the required amount of travel under the different spatial distributions of workers and jobs in each city. The first measure is found to be the average trip distance.
Figure 3. Differential Accumulation of Population vs. Jobs, in percent, by Distance from the City's Center.
2.6 The importance of the d/a process is that it relates, in a simple way, the two spatial distributions of workers and jobs to a most elementary descriptor of travel behavior, namely the average journey-to-work distance ($d_w$), by the formula:

$$d_w = \frac{\sum_{i=1}^{n} x_i g(x_i)}{\sum_{i=1}^{n} g(x_i)} ;$$  \hspace{1cm} (2.1)

App. 1 presents a comparison between the estimated average trip distance to work by Eq. 2.1 and the observed average trip distance in a selection of cities, where the close similarity between the two sets is strongly evident.

It seems that such accuracy cannot be solely due to chance, especially since the estimated average trip distances to work are based on several assumptions and approximations (such as radial symmetry of the two distributions around the center) while the observed average trip distances are for all daily trips, to all purposes, after linking the trips by various definitions and procedures, depending on the study. Indeed, the d/a process can be restated in more familiar terms, as detailed in the next section.

The strong connection between the average trip distance and urban structure was discussed in many previous studies (8,9), focusing mainly on cross sectional observations by which the former was explained by the latter. It now appears that the interaction mechanism between the two is more complex, namely that both closely interact over time, where the trip distance is more the cause of, rather than the effect from changes in urban structure.

2.7 The d/a process can be restated in the following way:

Let $p(x)$ and $e(x)$ be the continuous density distributions of workers and jobs as a function of distance from the center of the city. No assumptions about the distributions $p(x)$ and $e(x)$ are made, other than:

$$\int_{0}^{L} p(x)dx = \int_{0}^{L} e(x)dx = 1 ;$$  \hspace{1cm} (2.2)

Define the function $g(x)$ by:

$$g(x) = \int_{x}^{L} p(t)dt - \int_{x}^{L} e(t)dt ;$$  \hspace{1cm} (2.3)

in analogy with the earlier definition of $g(x_i)$. The first integral gives the number of workers that are located at a distance of at least $x$ length units from the center. The second integral gives the number of jobs at distance $x$ or more from the center. The difference, as before, is called the d/a function.

The first thing to observe about Eq. 2.3 is that there is nothing unique about starting with the outer ring. We could just as easily have gone in the other direction since

$$g(x) = \int_{x}^{L} p(t)dt - \int_{x}^{L} e(t)dt = \int_{0}^{x} e(t)dt - \int_{0}^{x} p(t)dt ;$$  \hspace{1cm} (2.4)
For convenience, we will also write \( g(x) \) in the equivalent form:

\[
g(x) = 1 - P(x) - (1 - E(x)) ; \tag{2.5}
\]

where \( P(x) = \int_0^x p(t) \, dt \) and \( E(x) = \int_0^x e(t) \, dt \). From this last, we obtain:

\[
\int_0^L g(x) \, dx = \int_0^L [1-P(x)] \, dx - \int_0^L [1-E(x)] \, dx = d_p - d_E ; \tag{2.6}
\]

where \( d_p \) and \( d_E \) are the mean distances of workers and jobs from the center.

Equation 2.6 says that the area under the \( d/a \) curve is the difference of mean distances of workers and jobs from the center.

2.8 The next term we need to evaluate it terms of the assumed (or observed) distributions \( p(x) \) and \( e(x) \) is \( \int_0^L x g(x) \, dx \).

From Eq. 2.5 we have:

\[
\int_0^L x g(x) \, dx = \int_0^L x [1-P(x)] \, dx - \int_0^L x [1-E(x)] \, dx \\
= \frac{1}{2} \int_0^L x^2 p(x) \, dx - \frac{1}{2} \int_0^L x^2 e(x) \, dx ;
\]

\[
\int_0^L x g(x) \, dx = \frac{1}{2} \left\{ m_2(P) - m_2(E) \right\} ; \tag{2.7}
\]

where we have written \( m_2(P) \) and \( m_2(E) \) for the second moments about the origin of \( p(x) \) and \( e(x) \), respectively.

From Eq. 2.6 and 2.7, we have

\[
\frac{\int_0^L x g(x) \, dx}{\int_0^L g(x) \, dx} = \frac{m_2(P) - m_2(E)}{2(d_p - d_E)} ; \tag{2.8}
\]

And from the observation that the mean of the normalized \( d/a \) function is a good estimate of the average journey-to-work distance \( d_w \), we have:

\[
d_w = \frac{m_2(P) - m_2(E)}{2(d_p - d_E)} ; \tag{2.9}
\]

The right-hand side of Eq. 2.9 can be rewritten in terms of the variances by recalling that:

\[
\sigma^2 = m_2 - d^2 .
\]
Thus,

\[
\frac{m_2(P) - m_2(E)}{2(d_P - d_E)^2} = \frac{\sigma_P^2 + \sigma_E^2 - \sigma_{PE}^2}{2(d_P - d_E)^2} = \frac{d_P + d_E}{2} + \frac{\sigma_P^2 - \sigma_E^2}{2(d_P - d_E)}; \tag{2.10}
\]

The first term on the right-hand side is simply the average of the distances of jobs and workers combined from the city center.

If we write

\[d = \frac{1}{2}(d_P + d_E)\]

and

\[\sigma = \frac{1}{2}(\sigma_P + \sigma_E),\]

we have

\[d = \frac{m_2(P) - m_2(E)}{2(d_P - d_E)} = d + \left[\frac{\sigma_P^2 - \sigma_E^2}{d_P - d_E}\right]\sigma; \tag{2.11}\]

This suggests that we may interpret the d/a process as saying:

The average journey-to-work distance can be approximated by \(d = \frac{1}{2}(d_P + d_E)\), where \(d_P\) is the average distance of workers' residences from the city center, and \(d_E\) is the average distance of job locations from the city center.

The approximation can be improved by adding an additional term given be

\[\frac{\sigma_P^2 - \sigma_E^2}{2(d_P - d_E)} \tag{2.12}\]

where \(\sigma_P^2\) and \(\sigma_E^2\) are the variances of the distribution of workers' residences and jobs from the city center, respectively.

2.9 The term \(d_P - d_E\), which we recall from Eq. 2.6 is the value of \(\int_0^L g(x)dx\), is subject to some interpretation. In particular, let us regard both workers and jobs as undifferentiated and distributed in the urban area in any way whatsoever, subject to the condition that if \(d_P\) and \(d_E\) are the average distances of the workers and jobs from the city center, respectively, then \(d_P > d_E\). Then, if we define the classical assignment problem of assigning workers to jobs (assumed equal in number) in such a way as to minimize the
total distance traveled, it can be shown that a lower bound for the
total travel distance is \( N(d_p - d_B) \), where \( N \) is the number of
workers (and jobs). Thus, the average travel distance at the lower
bound is given by \( d_p - d_B \), the area under the d/a function \( g(x) \).
This will clearly be no more than the minimum obtained if the
workers and jobs are differentiated.

It is of interest to note that the additional factor, expressed in
Eq.2.12, turns out to be relatively small, thus indicating that
workers tend to travel as near as possible to the theoretical
minimum distance between the spatial distributions of their resi-
dences and jobs. This result appears to corroborate previous
similar indications (10).

2.10 The normalized d/a function \( g^*(x) \), as shown in Figure 3, and
given by
\[
g^*(x) = g(x)/\int_0^L g(x)dx
\]
can be approximated by a Gamma distribution
\[
T(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{(\alpha-1)!};
\]  
(2.13)
The distribution is determined by the shape parameter \( \alpha \), and the
scale parameter \( \beta \). Both parameters can be estimated from finite
data by requiring that the Gamma distribution have the same mean
and second moments as the normalized d/a function. With a little
algebra, this produces:
\[
\alpha = \frac{\left[ \sum x_i g(x_i) \right]^2}{\sum g(x_i) \sum x_i^2 g(x_i) - \left[ \sum x_i g(x_i) \right]^2};
\]  
(2.14)
\[
\beta = \frac{\sum g(x_i) \sum x_i^2 g(x_i) - \left[ \sum x_i g(x_i) \right]^2}{\sum g(x_i) \sum x_i^2 g(x_i) - \left[ \sum x_i g(x_i) \right]^2};
\]  
(2.15)

Thus, \( \alpha \) and \( \beta \) are determined by \( \sum g(x_i), \sum x_i g(x_i) \) and \( \sum x_i^2 g(x_i) \),
the approximate zeroth, first and second moments of the d/a function,
all readily calculated from the data.

It can be further shown that the ratio \( \alpha/\beta \) expresses the average
distance of the d/a curve from the city center, thus \( \alpha/\beta \) approximates
the average trip distance in the city. This, indeed, seems to be the
case for the selection of cities shown in Figure 3. Thus, a direct link between the average trip distance and descriptors of the urban structure can be established.

Furthermore, the parameters $a$ and $b$ make it possible to rank cities by their internal structure, as expressed by the spatial distributions of population and jobs. The ranking of the cities given in Figure 3 is shown in Figure 4.

3. **INTERPRETATIONS AND FURTHER POSSIBILITIES**

3.1 There are several possible ways of interpreting the d/a process. The one preferred here is the explanation that seems to tie up travel behavior, system supply and urban structure, as follows:

1. It was already indicated above that tripmakers strive to maximize their daily travel distance within the double constraints of their household’s travel money budget and their own travel time budget;

2. Furthermore, it was also indicated that the trip distance and the trip rate (the product of which equals the total daily travel distance) are closely interlinked by the perceived value of each, as expressed by their cross elasticities vs. changes in the daily average travel speed;

3. Hence, if the city is large and the speed is low, the need to make a minimum daily number of trips per tripmaker will force him to economize on his trip distance. As a result, it is to be expected that the differentiation between the spatial distributions of population and jobs will be relatively low for a given size of such a city. Such cities are mostly found in developing countries, and Bangkok can be cited as an example, where the two distributions are relatively evenly spread out in the urban area;

4. When the average speed of travel, weighted by all tripmakers, increases by an increase in the motorization level, by an improvement in the transportation system, or by both, an increase in the average trip distance is to be expected, proportional to the cross elasticities between trip distance and trip rate vs. speed, as well as a rise in the shape of the d/a curve. As a result, the differentiation between the two distributions is expected to increase as well, with a dispersion of population and jobs from the city center (depending, of course, also on the amount of land use control exercised in the area, or the availability of land, a subject that will be discussed later). The case of Washington, D.C. during 1955-1968 can be cited as an example;

5. Conversely, if speeds decrease, such as by congestion, tripmakers will have to economize on their trip distance and, hence, decrease the differentiation between the two distributions, thus initiating a process of job dispersion. Furthermore, if speeds will decrease at and near the center, while they will increase at the periphery (such as by a beltway), both processes may take place
1. Athens, 1962
2. Baltimore, 1962
3. Bangkok, 1972
4. Bristol, 1966
5. Kuala Lumpur, 1973
7. Tel Aviv, 1965

Figure 4: Ranking Cities by the Parameters of the Gamma Function. (Ref. 10).
simultaneously, namely the dispersion of both population and jobs, at an accelerated pace;

(6) In relatively small cities, where the trip distance is short and
the speed is relatively high, the differentiation between the two
distributions is expected to be higher than in large cities, even
if the socio-economic characteristics are relatively similar in both
city categories. Bristol and London can be cited as examples;

(7) The inference from all the above is that the spatial structure
of a city, as expressed by the spatial distributions of population
and jobs, can be linked to travel behavior and system supply be feed
back loops, as shown in Figure 1.

3.2 As already emphasized earlier, the above mechanism represents
only one factor in the life of and activities in cities and, as such,
can only approximate the complex processes taking place within them.
Nonetheless, since travel is recognized to reflect to a large extent
the dynamic aspects of static distributions in a city (as expressed
in all urban models in one way or another), the first-approximation
estimation by the empirical d/a process seems to be no worse than the
estimations made by much more sophisticated and complicated processes,
most of which are based on assumed relationships that need lengthy
calibrations and require a multitude of detailed data for their appli-
cation. Moreover, the d/a process links travel demand, system supply
and urban structure, an advantage that may compensate its present
approximation capabilities. Hence, the d/a process can be applied
presently not only as a controlling measure for assessing the total
shifts and changes in land uses that result from more sophisticated
urban models, but it can also be further developed to higher levels of
sophistication. For instance, differentiation of the urban area
by sectors, differentiation of the workers by income, car availability,
white and blue collar jobs, and so on.

3.3 An additional potential development is the derivation of the rent
distribution in the urban area from the UMOT and the d/a processes
coupled with the constraints imposed on the expansion of the urban
area, either by geographic limitations or by policy restrictions,
as well as the rent changes brought by population increase under
various assumed urban developments.

3.4 The d/a process can also be used to test alternative land use
plans and their possible effects on the amount of travel that they
will generate and, hence, the amount of system supply that they will
require.

Such an example is shown in Figure 5, for three alternative land use
plans for Bangkok (11). Plan G calls for the concentration of commer-
cial land uses and activities in and around the city center, and the
dispersion of population outwards; Plan U projects the natural past
trends of growth into the future; and Plan F suggests a polycentric
development, based on a balanced dispersion of both population and
jobs.

As can be seen in Figure 5, Plan G appears to put the heaviest burden
of travel on the population, requiring an extensive development of
Figure 5. Differential Accumulation of Population vs. Jobs by Distance from the City's Center, Bangkok-1972 and 1990 Alternatives
the transportation network, of both roads and transit, while Plan P appears to be the best of the three alternatives, traffic-wise. (It should be noted that Figure 5 presents population values normalized to the absolute number of jobs, thus the amounts of travel are proportional to the expected travel to work).

An additional interesting aspect is that both Plans U and P indicate the start of regional development, where a significant proportion of jobs is to be located at the fringe of the urban area, thus attracting part of the travel outwards, rather than having all work trips moving towards the center. This trend is indicated by the negative values of the d/a curves near the fringe of the city. Hence, the d/a process can also assist in delimiting the urban area, traffic-wise, from the regional area.

In the case of Bangkok the d/a process also produced estimates of the average trip distance for each alternative, that were practically identical with the estimates produced by a comprehensive traffic model. All in all, therefore, the actual application of the d/a process was found to be very encouraging.

3.5 An additional test for the d/a capabilities was to estimate the spatial distributions of population and jobs in Washington, D.C. 1968, when based on only the 1955 given two distributions, the 1968 number of households by income, and the assumption that households owning cars (expressed by their income level in 1968) will succeed in bringing their travel demand into equilibrium conditions with system supply. The resulting estimates of the two spatial distributions for 1968, including the expansion of the urban area under no land use restrictions, were surprisingly similar to the observed distributions.

3.6 It becomes apparent from all the above indications that the transportation system supply has a major effect on urban structure, within travel behavior and the given policies on the level of land use control. Namely, increasing the speed of travel by improving the system supply will generate more (induced) daily travel, that will tend to increase both the trip distance and the trip rate, with the tripmakers preference directed towards the former. Hence, if the city will be allowed to expand freely, a process of population dispersion will start, where most of the benefits will be directed towards better spatial opportunities between pairs of origin-destination, including residence-work pairs. If, on the other hand, the expansion of the city will be restricted by land use control (especially at the periphery of the city), then a significant part of the speed increase will be traded off for more trips, thus changing the proportions of trip purposes towards more other-than-work trips, such as social-recreation trips, with a possible increase in the social interaction within the urban area, and with less income polarization.

The effect of system supply on changes in the spatial distributions of population and jobs has been known for a very long time. The d/a process, however, enables now to explain and forecast such changes in quantified terms by a simple process, as well as show the way how to channel the physical changes of a city in the direction preferred by the policy makers.
IN CONCLUSION

The empirical relationship of the differential accumulation between the spatial distributions of population and jobs is an intriguing one. Even in its simplistic form it appears to produce, time and again, estimates of travel measures that are very close to the observed ones, as derived by complex, lengthy and costly travel surveys and model calibrations. Hence, it can be used, even in its present early development form, as a useful tool for rapid assessment of policy decisions on both system supply and urban structure, and the possible impacts of each on the other.

Another advantage of the d/a process is that it appears to be fully transferable between cities and, hence, seems to be applicable also for one city over time.

Nonetheless, the d/a process is in the early stages of development, and much more search and research are still needed before it can be regarded as fully verified. It is suggested, therefore, that the future development of the d/a process will proceed in two avenues: to verify its validity in many more additional cities, and for micro-conditions within each city; and to further develop its theoretical foundation on behavioral grounds.

* * *

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### App. 1. Estimated vs. Observed Car Average Trip Distance, km.

<table>
<thead>
<tr>
<th>City</th>
<th>Year</th>
<th>$\sum x_i g(x_i)$</th>
<th>$\sum g(x_i)$</th>
<th>$\frac{\sum x_i g(x_i)}{\sum g(x_i)}$</th>
<th>d Obs.</th>
<th>Diff. $\frac{d}{d}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens</td>
<td>1962</td>
<td>301.3</td>
<td>54.5</td>
<td>5.5</td>
<td>5.6</td>
<td>-1.8</td>
</tr>
<tr>
<td>Kuala Lumpur</td>
<td>1972</td>
<td>457.4</td>
<td>90.6</td>
<td>5.1</td>
<td>5.3</td>
<td>-3.8</td>
</tr>
<tr>
<td>Tel Aviv</td>
<td>1965</td>
<td>335.3</td>
<td>81.6</td>
<td>4.1</td>
<td>4.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Bangkok</td>
<td>1972</td>
<td>305.7</td>
<td>42.1</td>
<td>7.3</td>
<td>7.4</td>
<td>-1.4</td>
</tr>
<tr>
<td>Baltimore</td>
<td>1962</td>
<td>667.0</td>
<td>73.8</td>
<td>9.0</td>
<td>9.3</td>
<td>-3.2</td>
</tr>
<tr>
<td>Washington, DC</td>
<td>1955</td>
<td>7,810,403</td>
<td>1,176,647</td>
<td>6.64</td>
<td>6.68</td>
<td>-0.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>1968</td>
<td>22,814,018</td>
<td>2,258,634</td>
<td>10.10</td>
<td>10.62</td>
<td>-4.9</td>
</tr>
</tbody>
</table>
REFERENCES


