TO: Mr. Harold B. Dunkerley
FROM: Yacov Zahavi (Consultant)

SUBJECT: The Unified Mechanism of Travel (UMOT) Model

CONTENTS

Preface and Summary .............................................. Page 3

I. The Travel Cost Budget ........................................ 7
   1.1 Introduction ............................................. 7
   1.2 Travel cost budgets in the US, 1963 - 1973 ............. 7
   1.3 Travel cost budgets in the UK ........................... 8
   1.4 Travel cost budgets in London ........................... 10
   1.5 Travel cost budgets in Washington, DC ................. 11
   1.6 Travel cost budgets in Kuala Lumpur .................... 12
   1.7 The effect of the travel cost budget on travel distance 12
   1.8 The travel cost budget per tripmaker ................... 14
   1.9 In conclusion ............................................ 15

II. The Travel Time Budget ......................................... 17
   2.1 Introduction ............................................. 17
   2.2 The TT-budget ............................................. 18
   2.3 The effect of speed on the TT-budget .................... 19
   2.4 The effect of speed on the daily travel distance ....... 20
   2.5 The effect of speed on trip distance and trip rate .... 21
   2.6 Travel time budgets in cities of developing countries 21
   2.7 In conclusion ............................................ 22

III. Equilibrium Conditions of Travel .......................... 23
   3.1 Introduction ............................................. 23
   3.2 Equilibrium between the two travel budgets ............ 23
   3.3 The cost per unit distance of travel in the US ......... 24
   3.4 The travel cost budget in Washington, DC, 1968 ....... 26
   3.5 Travel equilibrium in Washington, DC, 1968 ............ 27
      (1) Equilibrium between the two travel budgets ........ 27
      (2) Estimation of travel by the UMOT model ............ 28
      (3) Estimation of modal split by distance and by trips 30
   3.6 The effect of travel equilibrium on residence location 32
   3.7 Elasticity of travel distance vs travel cost .......... 34
   3.8 Travel equilibrium under policy changes ............... 36
   3.9 The effect of travel cost on motorization levels ...... 38
   3.10 In conclusion ............................................ 39
IV. Travel in Cities of Developing Countries

4.1 Introduction ........................................... 41
4.2 Unit Cost of travel in Kuala Lumpur .............. 41
4.3 Private vehicle-miles of travel, K.L. 1972 .... 42
4.4 Person-miles of travel by mode .................... 43
4.5 Person trips by mode .................................. 43
4.6 Estimating the travel costs in K.L. ................. 44
4.7 In conclusion ........................................... 46

V. The Urban Structure

5.1 Introduction ........................................... 47
5.2 The differential accumulation process ............. 47
5.3 Estimating average trip distances by the D.A. function 48
5.4 The D.A. function and city structure ............... 49
5.5 Application of the D.A. function ..................... 51
5.6 In conclusion ........................................... 54

VI. Structure of the UMT Model

6.1 Introduction ........................................... 57
6.2 Inputs and outputs .................................... 57
6.3 Inputs subject to policy changes .................... 58
6.4 Feedback iterations .................................. 59
6.5 The perceived value of trip rate .................... 59
6.6 In conclusion ........................................... 61

VII. Conclusions and Recommendations

7.1 The UMT vs. other Models ......................... 63
7.2 Recommendations ..................................... 64

References ................................................. 66

Appendices ................................................ 67

1. Unit Cost of Car Travel in the U.S.
2. Cost of Car Travel in Washington, D.C. 1968
4. Person Miles of Travel by District, Washington, D.C. 1968
5. Estimation of Travel in Kuala Lumpur, 1972
6. Two Reports by Mr. James M. McLynn:
   6.2. The Differential Accumulation Process
7. The Differential Accumulation Technique
8. Travel Demand vs. System Supply, K.L. 1972 and 1980
PREFACE AND SUMMARY

1. This report suggests a unified mechanism of travel (UMOT, for short), where travel demand, transportation system supply and urban structure interact, striving to reach equilibrium under continuously changing travel and urban conditions.

2. This report complements and summarizes previous reports on the same subject, as follows:

   (1) The spatial transportation model - October 13, 1975;
   (2) City structure and mobility - November 7, 1975;
   (3) The value of mobility - January 30, 1976;
   (4) A direct procedure for measuring the value of mobility and benefits from improvements in transportation systems - February 17, 1976;

3. In its present form, the UMOT model is macro in form, supplying estimates on travel and urban structure from the level of controlling totals down to the level of districts. Although there are no limitations on developing it to the level of traffic zones, its ability to represent micro travel conditions has still to be verified by actual tests.

4. The principal findings while developing the UMOT model may be summarized as follows:

   (1) Households are willing to allocate a certain proportion of their income to travel, a proportion that is relatively stable both between cities and over time in developed countries;
   (2) The daily expenditure on travel per tripmaker increases as income increases, but at decreasing rates;
   (3) The daily travel time per tripmaker in developed countries tends to be stable both between cities and over time;
   (4) The analysis of the available data suggests that tripmakers tend to maximize their travel distance within the constraints of their daily travel money and time budgets;
   (5) The fundamental equation of travel in its simplified form states that the household's value of travel time, expressed as quotient of the travel cost budget (Exp.) over the travel time budget (T), equals the product of the speed of travel (v) and the cost per unit distance travelled (c). Namely:

       \[ \frac{\text{Exp.}}{T} = v \cdot c \]

   While the left hand side of the equation represents travel demand in terms of cost and time budgets, the right hand side represents the product that the household would like to purchase from the transportation system supply, in terms of the system's performance and the price of using it.
(6) Increases in income result in increasing demand for speed. Since the unit cost of travel decreases with increases in speed, within the speed range found in cities, the total cost of travel increases with distance, but at decreasing rates, thus serving as an incentive for increasing the spatial opportunities in the city;

(7) Households strive to reach equilibrium conditions between their travel demand and system supply by adjusting their amount and spatial patterns of travel. Hence, system supply will affect the household's residence location, amount of travel distance, choice of travel mode, and patterns of O-D distribution;

(8) Households owning a car are found to have better opportunities for equalizing their travel demand within their money and time budgets. Conversely, households not owning a car that have to travel by the slower transit, expend their travel time budget much before they reach even half of their travel cost budget, and the higher the income - the farther are they from reaching equilibrium conditions, resulting in an increasing incentive to travel by car.

(9) The households' efforts to reach equilibrium conditions between their travel demand, as expressed by their travel budgets, and system supply, also result in shifts and changes in the land use patterns. The spatial distributions of population and jobs interact in such a way as to result in a predictable average trip distance in the city;

(10) Furthermore, tripmakers are found to prefer to trade off increases in speed for longer trip distances than for more trips. Hence, changes in the system supply become one of the major forces to reshape the city structure and may, therefore, be used as a lever for desired changes in the urban form;

(11) Although there are no available data on the travel cost and time budgets in cities of developing countries, there are enough indications to suggest that both budgets exceed the corresponding budgets in cities of developed countries, by about fifty percent. There are two main possible explanations for this unexpected result: (i) travel speeds are low and, therefore, the unit costs of travel are high. Hence, tripmakers have to pay more heavily in both money and time in order to travel the same travel distance per day; (ii) the required daily travel distance is higher than expected in relatively dense cities because the urban structure is far from being in equilibrium with system supply and travel demand, continuously expanded by poor migrants settling at the periphery.

(12) The travel situation in such cities is even further aggrevated by two additional factors: (i) the poor newcomers, settling down peripherically, have the farthest to travel to jobs, although they are the most constrained in their travel cost budget; (ii) in some cities, a relatively high proportion of the population cannot even afford a bus fare, and have to walk. Hence, the latent, potential, travel demand is hard to define from the standard transportation surveys;

(13) All the above difficulties, found in cities of developing countries, have raised doubts about the ability of the UMOT model, which was developed from detailed data relating to U.S. cities, to reflect and simulate travel conditions in cities of developing countries. It is of interest to note, therefore, that for the one
city for which there were enough test data, Kuala Lumpur, the estimated and the observed travel characteristics for all travel parameters match to within a few percent. Many more such tests will have to be made before the UMOT model can be regarded as a reliable tool for analysis and planning of transportation and urban alternatives in cities of developing countries but, even at this early stage of development, the UMOT model has shown its ability to be applied to such diversified cities as Washington, DC and Kuala Lumpur with equally satisfactory results.

(14) The analyses during the development of the UMOT have strongly indicated that the present procedures for transportation and urban planning should be reevaluated in light of the following findings: (i) trip-makers strive to maximize their travel cost and time budgets. Thus, in response to changes in the transportation system supply, they change their travel patterns, as well as the urban structure, in order to reach an equilibrium condition between travel demand, system supply and urban structure; (ii) cities in developing countries are far from being in an equilibrium condition, thus imposing on their population extremely high burdens in terms of money and time travel expenditures; (iii) any urban project, such as housing and job development should, therefore, be carefully assessed in relation to its transportation implications; (iv) conversely, transportation planning for such cities may be used as a lever for leading the development of the city in the desired direction.

(15) The report concludes with some thoughts on new methods for the economic evaluation and justification of transportation and urban projects, as well as recommendations for further development of the UMOT model. Foremost are the following recommendations: (i) establish with more precision the equilibrium conditions between the money and time budgets for travel by all income levels in cities of developing countries; (ii) while the income threshold for car ownership has already been defined with satisfactory accuracy, there is now an urgent need to define the income thresholds between walking, cycling and using public transport, in order to estimate the latent travel demand of poor people; (iii) an additional factor that has to be introduced into the UMOT model is the dynamic time lag between the introduction of a transportation improvement and the resulting changes in land uses, in order to be able to assess the best phasing of the transportation and the urban plans.

5. This report is presented in draft form, with the aim of amending and supplementing it in the light of the readers' comments and advice. Such comments will, therefore, be much appreciated. The structure of the report is based on tables and diagrams with supporting text, while most of the calculations are detailed in appendices.

6. Special thanks are extended to Messrs. G. J. Roth and A. A. Walters, for their review of some parts of the early analyses; to J. Mclynn for his vigorous mathematical analysis of intriguing and elusive relationships, as presented in the appendices; and to the Transportation and Urban Projects Department of the World Bank, for continued encouragement and support.
1. The Travel Cost Budget

1.1 Introduction

The cost of travel is recognized to be a major component of, and a constraint on, travel. Since people have to allocate their disposable income to competing purposes, according to their perceived priorities, it follows that transportation will be assigned a certain proportion of their income.

This chapter will show how the cost of travel can be applied as a major constraint on travel at all levels, from a controlling total for the whole urban area and down to single trips within it. It will be shown that there seems to be a travel cost budget, strongly related to income $I$, which affects the quantity and quality of generated travel by tripmakers, and which follows established economic relationships. It will also be shown how the cost per unit distance travelled by different modes strongly affects the amount of travel, leading to a procedure by which changes in the amount of travel can be estimated for changes in unit costs. The chapter starts with broad-brush comparisons, followed by increasingly detailed relationships.

1.2 Travel Cost Budgets in the US, 1963-1973

Table 1.1 details the average personal consumption expenditure on travel versus the total consumption expenditure in all the US during 1963-1973 (1).

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Expenditure (Million $)</th>
<th>Exp. on Travel (Million $)</th>
<th>Exp. as % of Total Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963</td>
<td>374,982</td>
<td>49,110</td>
<td>13.1</td>
</tr>
<tr>
<td>1</td>
<td>401,356</td>
<td>51,753</td>
<td>12.9</td>
</tr>
<tr>
<td>5</td>
<td>431,665</td>
<td>57,825</td>
<td>13.4</td>
</tr>
<tr>
<td>6</td>
<td>466,334</td>
<td>60,689</td>
<td>13.0</td>
</tr>
<tr>
<td>7</td>
<td>492,066</td>
<td>62,588</td>
<td>12.7</td>
</tr>
<tr>
<td>8</td>
<td>536,178</td>
<td>71,873</td>
<td>13.4</td>
</tr>
<tr>
<td>9</td>
<td>579,457</td>
<td>77,772</td>
<td>13.4</td>
</tr>
<tr>
<td>1970</td>
<td>617,614</td>
<td>77,776</td>
<td>12.6</td>
</tr>
<tr>
<td>1</td>
<td>667,125</td>
<td>90,689</td>
<td>13.6</td>
</tr>
<tr>
<td>2</td>
<td>729,017</td>
<td>99,949</td>
<td>13.7</td>
</tr>
<tr>
<td>3</td>
<td>805,221</td>
<td>109,226</td>
<td>13.6</td>
</tr>
</tbody>
</table>

As can be seen, the expenditure on travel tended to be a stable proportion of the total expenditure, at about 13.2 percent. Since the total expenditure was found to be about 66 percent of income, it follows that the average expenditure on travel during 1963-1973 was about 11.1 percent of income.

(1) Because of lack of detailed data on disposable incomes, the analysis had to be limited to total incomes, although further refinements of the UMOT model will have to consider disposable incomes.
1.3 Travel Cost Budgets in the UK

Table 1.2 details the average household's weekly expenditure on travel versus the household's weekly income in all the UK in 1972 (2).

Table 1.2: Household Expenditure on Travel vs. Household Income, All U.K. (1972)

<table>
<thead>
<tr>
<th>HH Weekly Income, L</th>
<th>HH Weekly Expenditure on Travel</th>
<th>Exp. as % of Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>0.25</td>
<td>3.3</td>
</tr>
<tr>
<td>10-15</td>
<td>0.67</td>
<td>5.4</td>
</tr>
<tr>
<td>15-20</td>
<td>1.13</td>
<td>6.5</td>
</tr>
<tr>
<td>20-25</td>
<td>2.26</td>
<td>10.1</td>
</tr>
<tr>
<td>25-30</td>
<td>3.26</td>
<td>11.9</td>
</tr>
<tr>
<td>30-35</td>
<td>3.55</td>
<td>10.9</td>
</tr>
<tr>
<td>35-40</td>
<td>4.27</td>
<td>11.4</td>
</tr>
<tr>
<td>40-45</td>
<td>4.61</td>
<td>10.9</td>
</tr>
<tr>
<td>45-50</td>
<td>5.74</td>
<td>12.1</td>
</tr>
<tr>
<td>50-60</td>
<td>7.24</td>
<td>13.2</td>
</tr>
<tr>
<td>60-80</td>
<td>8.60</td>
<td>12.3</td>
</tr>
<tr>
<td>80 plus</td>
<td>12.77</td>
<td>12.8</td>
</tr>
<tr>
<td>(12.5)</td>
<td>4.97</td>
<td>(11.7)</td>
</tr>
</tbody>
</table>

(20,472 Households)

Figure 1.1 expresses Table 1.2 in a graphical form, where it becomes evident that the expenditure on travel tends to increase linearly with increasing incomes within the range of available data. Percentagewise, the expenditure tends to increase rapidly only at very low income levels, levelling off at about 10-13 percent along most income levels, with a weighted average value of about 11.7 percent. Namely, practically identical with the proportion found in the US.
Figure 1.1: Household Weekly Expenditure on Travel vs. Household Weekly Income, U.K. (1972)
1.4 Travel Cost Budgets in London

Table 1.3 presents the average household's weekly expenditure on travel versus the household's weekly income in London, 1972 (2).

Table 1.3: Household Expenditure on Travel vs. Household Income, London (1972)

<table>
<thead>
<tr>
<th>HH Weekly Income £</th>
<th>HH Weekly Expenditure on Travel</th>
<th>Exp. as % of Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-15</td>
<td>0.37</td>
<td>3.7</td>
</tr>
<tr>
<td>15-30</td>
<td>1.34</td>
<td>6.0</td>
</tr>
<tr>
<td>30-40</td>
<td>4.04</td>
<td>11.5</td>
</tr>
<tr>
<td>40-50</td>
<td>4.64</td>
<td>10.3</td>
</tr>
<tr>
<td>50-60</td>
<td>9.09</td>
<td>16.5</td>
</tr>
<tr>
<td>60 plus</td>
<td>10.51</td>
<td>13.1</td>
</tr>
<tr>
<td>(£68.5)</td>
<td>5.98</td>
<td>(12.3)</td>
</tr>
</tbody>
</table>

(850 Households)

Figure 1.2, based on table 1.3, shows the same trends as the nationwide ones, namely a linear increase of expenditure with income, with a weighted average value of about 12.3 percent.

Figure 1.2: Household Weekly Expenditure on Travel vs. Household Weekly Income, London, 1972.
1.5 Travel Cost Budgets in Washington, DC

While the previous examples were based on data derived directly from household surveys on actual expenditures, the available data from the 1968 transportation study in Washington, DC included information on incomes only (l). Hence, a special analysis on the unit costs of travel in 1968, as presented in Appendix 1, had to be carried out while developing and testing the UMOT model.

Table 1.4 and Figure 1.3 present the estimated expenditures on travel in Washington, DC, 1968, based on the travel distance per household at different income levels and the unit costs of travel, while the detailed calculations are presented in Appendix 2.

Table 1.4: Household Daily Expenditure on Travel vs. Household Annual Income, Washington, D.C. 1968 (within 1955 Gordon)

<table>
<thead>
<tr>
<th>HH Annual Income, $</th>
<th>HH Daily Expenditure on Travel (312 days)</th>
<th>Exp. as % of Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,000</td>
<td>1.01</td>
<td>5.3</td>
</tr>
<tr>
<td>7,000</td>
<td>1.85</td>
<td>8.2</td>
</tr>
<tr>
<td>8,000</td>
<td>2.67</td>
<td>10.4</td>
</tr>
<tr>
<td>9,000</td>
<td>3.36</td>
<td>11.7</td>
</tr>
<tr>
<td>10,000</td>
<td>3.91</td>
<td>12.3</td>
</tr>
<tr>
<td>11,000</td>
<td>4.45</td>
<td>12.6</td>
</tr>
<tr>
<td>12,000</td>
<td>4.97</td>
<td>12.9</td>
</tr>
<tr>
<td>13,500</td>
<td>3.05</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Once again, it becomes evident that the expenditures on travel tend to follow consistent trends, with a weighted average value of about 11.2 percent of income.

Figure 1.3: Household Daily Expenditure on Travel vs. Household Annual Income, Washington, D.C. 1968
1.6 Travel Cost Budgets in Kuala Lumpur

The data from the Kuala Lumpur transportation study, 1973 (5), as in Washington, DC, included data on incomes, but not on expenditures. In this case, however, estimated unit costs of travel were available (6), and Table 1.5 is based on these estimates.

Table 1.5: Household Daily Expenditure on Travel vs. Household Monthly Income, Kuala Lumpur, (1973)

<table>
<thead>
<tr>
<th>Vehicle Availability</th>
<th>HH Monthly Income, M$</th>
<th>HH Daily Exp. on Travel, M$ (312 days)</th>
<th>Exp. as % of Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>No vehicle</td>
<td>338</td>
<td>0.98</td>
<td>7.5</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>416</td>
<td>2.09</td>
<td>13.1</td>
</tr>
<tr>
<td>1 Car</td>
<td>856</td>
<td>5.27</td>
<td>16.0</td>
</tr>
<tr>
<td>2+ Cars</td>
<td>1,690</td>
<td>9.46</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>598</td>
<td>2.92</td>
<td>12.8</td>
</tr>
</tbody>
</table>

As can be seen, the results follow the previous examples, where expenditures on travel increase with income, and tend to be stabilized at a weighted average value of 12.6 percent.

Up to this point, all examples, both in cities of developed and developing countries, seem to be practically identical. However, a careful comparison between the unit costs of travel in Kuala Lumpur and the unit costs in the US has indicated that the former are underestimated by a considerable amount. Furthermore, and as will be discussed in detail later, the longer daily travel times of tripmakers in cities of developing countries, as well as their higher daily trip rates, suggest that expenditures on travel in these cities, as a proportion of income, are much higher than expected. Hence, Table 1.5 will be reevaluated later, together with several possible explanations for the unexpectedly high expenditures on travel in cities of developing countries.

1.7 The Effect of the Travel Cost Budget on Travel Distance

When incomes increase, expenditures on travel increase linearly with them within the range of the available data. The most noticeable effect of such increases is a parallel increase in the daily distance travelled. Table 1.6 and Figure 1.4 summarize the daily travel distance per household vs income for the cases mentioned above, where it becomes evident that increases in income also increase the spatial opportunities of the households, by allowing them to cover longer travel distances during a day.

Table 1.6: Household Travel Distance vs. Household Income

<table>
<thead>
<tr>
<th>HH Yearly Income, £</th>
<th>HH Daily Travel, Km.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All U.K., 1972</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>11.8</td>
</tr>
<tr>
<td>500-750</td>
<td>13.7</td>
</tr>
<tr>
<td>750-1,000</td>
<td>23.3</td>
</tr>
<tr>
<td>1,000-1,500</td>
<td>36.5</td>
</tr>
<tr>
<td>1,500-2,000</td>
<td>49.7</td>
</tr>
<tr>
<td>2,000-3,000</td>
<td>61.0</td>
</tr>
<tr>
<td>3,000-4,000</td>
<td>82.5</td>
</tr>
<tr>
<td>4,000 plus</td>
<td>96.9</td>
</tr>
<tr>
<td>City</td>
<td>Car Availability</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Washington</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2+</td>
</tr>
<tr>
<td>Twin Cities</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2+</td>
</tr>
</tbody>
</table>

Figure 1.4: Daily Distance Traveled per Household vs. Household Annual Income, U.K. (1972), Washington, D.C. (1968) and Twin Cities (1970)
It will be shown later that the interaction between income, travel cost budget, cost per unit distance and the total distance travelled, is such that households at increasing incomes have a strong incentive to travel longer, since the cost per distance covered increases at decreasing rates. Put in another way, richer people can purchase superior products at relatively cheaper prices, while poorer people are forced to purchase inferior products at relatively high prices. This general observation seems to apply to transportation as well, and its possible implications to travel modelling and planning procedures will be discussed later.

1.8 The Travel Cost Budget per Tripmaker

Although it is the household budget that constrains the travel of household members, it is the tripmaker who decides whether or not to make a trip, to what purpose, by what mode, at what time, and to what destination. Hence, the household's travel cost budget will constrain the travel of the single tripmaker. While the relationship between the number of tripmakers per household vs the household's income and motorization levels will be discussed later on, it can be shown at this stage that the allocated travel cost budget per tripmaker tends to increase with the household's income at decreasing rates. Since data on tripmakers were available for only Washington, DC (and Twin cities), Table 1.7 and Figure 1.5 present the results for one case only. Nonetheless, the relationship follows the expected trend, namely, that expenditures should increase with income at decreasing rates.

<table>
<thead>
<tr>
<th>HH Annual Income, $</th>
<th>Household Daily Exp. $</th>
<th>Tripmaker per Household</th>
<th>Exp. per Tripmaker, $</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>2.08</td>
<td>1.37</td>
<td>1.52</td>
</tr>
<tr>
<td>7,000</td>
<td>2.60</td>
<td>1.56</td>
<td>1.65</td>
</tr>
<tr>
<td>8,000</td>
<td>3.13</td>
<td>1.79</td>
<td>1.75</td>
</tr>
<tr>
<td>9,000</td>
<td>3.56</td>
<td>2.00</td>
<td>1.79</td>
</tr>
<tr>
<td>10,000</td>
<td>4.04</td>
<td>2.21</td>
<td>1.83</td>
</tr>
<tr>
<td>11,000</td>
<td>4.51</td>
<td>2.42</td>
<td>1.86</td>
</tr>
</tbody>
</table>

Here, then, is an interesting phenomenon: while it should have been expected that the household's total expenditure on travel has to increase with its income at decreasing rates, the expected trend is found only at the level of the household's tripmakers. It may, therefore, be inferred that it is not the household, as such, which should be considered as the unit that generates travel, or that acts according to established economic criteria, but rather its tripmakers. Hence, the household's travel cost is the result of the aggregated travel behavior of its tripmakers, within the constraints of the household's income. Although this indication seems trivial and academic at this stage, it is an additional corroboration for regarding tripmakers, rather than households, as the building blocks of travel analysis and forecasting in the UMOT model.
1.9 In Conclusion

It may be inferred at this stage that:

1. Household's expenditures on travel tend to increase linearly with income within the range of available data;
2. Travel expenditures per tripmaker tend to increase with income at decreasing rates;
3. The weighted average expenditures on travel in countries and cities that are regarded as developed seem to be relatively stable, at about 11-13 percent of income. This trend suggests a relatively stable travel cost budget, which may be used as a controlling-total constraint for estimating the amount of travel generated under alternative system supply plans. It should, however, be noted that the above possibility does not necessarily imply that the travel cost budget has to remain constant, either for different transportation plans or over time; the only implication is that travel behavior on a daily basis (or, even better, on a monthly basis) has to be constrained by the expenditure that the household's members are willing, or able, to allocate for travel, with the expectation of deriving from it some benefit which is in some proportion to the benefits derived from other expenditures. Hence, the expenditures required to use alternative transportation plans would serve as

---

1/ An extreme example is a farmer in a developing country who will be willing to divert the transport of his goods to the market from horse-drawn carts to trucks; although the transport by trucks will be more expensive than by carts, he will be willing to do so with the expectation to increase his benefits, such as profits. Hence, an improved transportation does not necessarily imply savings in travel costs, or constant money expenditures on travel, but rather a relatively stable proportion of expenditure to income.
indications for the equilibrium condition of the urban transport market to satisfy such expenditures, leading to an additional method for evaluating the economic justification of transport projects.

Although the results in this chapter give a better insight into the mechanism of travel behavior, they also raise several new questions, such as: (i) are the low proportions of expenditures on travel at very low income levels the result of the low income only, or the result of additional factors? (ii) how does the household's income level affect the behavior of the individual tripmakers, such as mode choice or travel distance? or, conversely, how do the available modes, in a city affect the travel cost budget? (iii) what might be the effect of introducing a new mode, such as a rapid transit system, on the cost budgets and the amount of travel purchased by them?

However, before such questions can be answered, an additional travel budget will first have to be discussed, as detailed in the following chapter.

The last remark before closing this chapter is that although the empirical indications suggest that the travel cost budget is relatively stable as a proportion of income, and although it is treated in this report as independent of other cost budgets, it should, nonetheless, be regarded - and probably treated - as one component only of the household's total money budget.
2. The Travel Time Budget

2.1 Introduction

Transportation survey data have indicated that the daily travel time per tripmaker tends to be stable between cities in the same country, as well as over time. Thus, it was suggested that tripmakers tend to have a travel time budget (TT-budget, for short), in addition to their travel cost budget.

The TT-budget concept has been somewhat misunderstood, as some people assume that it is a constant value between cities and over time. However, the concept simply states that since all people have only 24 hours per day, they tend to divide the time between their various daily activities according to a ranking scale, similar to their money budgets. Thus, the time allocated in the US to travel is found to be about 1.1 hour per tripmaker, including access time (door-to-door travel time). Furthermore, since all people, rich and poor alike, have the same 24 hours per day at their disposal, it is to be expected that the variations in the TT-budget, when aggregated by zones or by socio-economic characteristics, should be smaller than the variations found in other budgets, such as the travel cost budget. Indeed, this seems to be the case.

In summary, while each individual may have his own ranking of priorities, affecting both his travel cost and travel time budgets, when aggregated under some characteristic, the money and the time constraints on their travel become strongly evident.

This chapter will present several aspects of the TT-budget and its effects on travel, while the interaction between the time and the money budgets will be discussed in chapter 3.

2.2 The TT-budget

As already shown elsewhere (7), the travel time per tripmaker tends to be stable in the US, both between cities and over time. Table 2.1 summarizes these times for car tripmakers in Washington, DC and Twin Cities in two time periods, as well as for the whole US. As can be seen, the travel times are very stable.

<table>
<thead>
<tr>
<th>Study Area</th>
<th>Year</th>
<th>TT. hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington, D.C.</td>
<td>1955</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1.11</td>
</tr>
<tr>
<td>Twin Cities</td>
<td>1958</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>1970</td>
<td>1.13</td>
</tr>
<tr>
<td>All U.S.</td>
<td>1970</td>
<td>1.10</td>
</tr>
</tbody>
</table>

It is suggested, therefore, that the travel time per tripmaker can be considered as a travel time budget, in parallel to the travel cost budget.
2.3 The Effect of Speed on the TT-budget

Most of the traffic models are based on the concept that people tend to save travel times. This concept seems to be strengthened by the observations that tripmakers at increasing incomes are willing to expend more in order to save travel time, such as by using toll roads and bridges. Hence, one may reach the conclusion that the value of saved travel time is strongly related to income, or to its derivatives, such as the wage per working hour. However, most of the observations have been based on single occurrences, such as single trips and, therefore, do not necessarily represent travel behavior under daily travel constraints. Indeed, when travel characteristics are considered on a daily basis, it becomes evident that tripmakers do not save travel time, as such, but trade them off for more travel.

Table 2.2 and Figure 2.1 present the effects of speed, and increase in speed over time, on the TT-budget of tripmakers, using different modes of transport in the two U.S. cities.

Table 2.2: Daily Travel Time per Tripmaker vs. Speed
Washington, D.C. and Twin Cities

<table>
<thead>
<tr>
<th>City</th>
<th>Year</th>
<th>Car TT, hr</th>
<th>v.kph</th>
<th>Transit TT, hr</th>
<th>v.kph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington, D.C.</td>
<td>1955</td>
<td>(1) 1.09</td>
<td>18.8</td>
<td>(3) 1.27</td>
<td>10.7</td>
</tr>
<tr>
<td>&quot;</td>
<td>1968</td>
<td>(2) 1.11</td>
<td>23.3</td>
<td>(4) 1.42</td>
<td>10.0</td>
</tr>
<tr>
<td>Twin Cities</td>
<td>1958</td>
<td>(5) 1.11</td>
<td>21.5</td>
<td>(6) 1.05</td>
<td>12.0</td>
</tr>
<tr>
<td>&quot;</td>
<td>1970</td>
<td>(6) 1.13</td>
<td>26.5</td>
<td>(6) 1.15</td>
<td>12.1</td>
</tr>
</tbody>
</table>

As can be seen, the TT-budgets of tripmakers, using car travel only, transit travel only and mixed modes, tend to be the same for all speeds down to a (door-to-door) speed of about 7.5 mph. This speed may be regarded as a critical minimum speed, below which tripmakers have to use an abnormal amount of travel time in order to make just 2 trips per tripmaker per day. Following the same trend, it may also be inferred that there should also be a second critical speed, this time a maximum speed, above which tripmakers would start to save travel time instead of travelling longer distances. However, it seems that the speeds found in the two US cities are still below this maximum speed.

Figure 2.1: Daily Travel Time per Tripmaker, by Mode, vs. Door-to-Door Speed
2.4 The Effect of Speed on the Daily Travel Distance

Since the travel time per tripmaker, under a wide range of speeds, tends to be stable, it follows that increases in speed should result in longer travel distances. Indeed, this trend is reflected in Table 2.3 and Figure 2.2, where the daily distance travelled per car tripmaker, stratified by traffic districts in Washington, D.C., tends to increase linearly with speed, both cross-sectionally and over time.

<table>
<thead>
<tr>
<th>District</th>
<th>1955</th>
<th>1968</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distance km</td>
<td>Speed kph</td>
</tr>
<tr>
<td>1</td>
<td>15.2</td>
<td>11.4</td>
</tr>
<tr>
<td>2</td>
<td>16.2</td>
<td>15.4</td>
</tr>
<tr>
<td>3</td>
<td>15.2</td>
<td>11.1</td>
</tr>
<tr>
<td>4</td>
<td>14.9</td>
<td>13.1</td>
</tr>
<tr>
<td>5</td>
<td>14.3</td>
<td>16.1</td>
</tr>
<tr>
<td>6</td>
<td>20.2</td>
<td>18.0</td>
</tr>
<tr>
<td>7</td>
<td>17.8</td>
<td>16.8</td>
</tr>
<tr>
<td>8</td>
<td>19.9</td>
<td>16.5</td>
</tr>
<tr>
<td>9</td>
<td>19.4</td>
<td>18.7</td>
</tr>
<tr>
<td>10</td>
<td>18.9</td>
<td>19.2</td>
</tr>
<tr>
<td>11</td>
<td>23.9</td>
<td>22.0</td>
</tr>
<tr>
<td>12</td>
<td>23.3</td>
<td>20.7</td>
</tr>
<tr>
<td>13</td>
<td>23.1</td>
<td>19.7</td>
</tr>
<tr>
<td>14</td>
<td>26.0</td>
<td>22.6</td>
</tr>
<tr>
<td>Total Average</td>
<td>20.5</td>
<td>18.8</td>
</tr>
</tbody>
</table>

Distance = 1.125 \times speed - 1.563 (1955 + 1968)

Coefficient of Correlation (r) = 0.94

Figure 2.2: Daily Distance Traveled per Tripmaker, by District, vs. Daily Average Door-to-Door Speed, Washington, D.C., 1968
In fact, all available data suggest that tripmakers prefer to trade off increases of speed for longer travel distance, rather than save travel time. Furthermore, and as will be shown later, longer travel distances entail higher total travel costs (although at decreasing rates). All indications, therefore, suggest that increases in travel speeds (such as brought by improving the transportation system) result not in savings in time and money, but rather in increases of the total travel costs within a stable TT-budget. In other words, tripmakers seem to be willing to pay more for a better product as long as they perceive it to increase their benefits.

The above result has some interesting implications to the interaction between the cost per unit travel distance and the speed of travel, as will be discussed in chapter 3. Let it only be noted at this stage that since the travel cost budget per tripmaker tends to increase with income at decreasing rates, while the travel distance per tripmaker tends to increase linearly with speed, it may be inferred that: (i) speed has to be related to income, and that (ii) the unit cost per distance has to decrease with increasing speed, within the speed range found in cities. These relationships suggest a possible mechanism of travel, where travel demand, as expressed by the travel money and time budgets, tends to purchase distance from the system supply, as expressed by the system's supply of speed at a certain cost.

2.5 The Effect of Speed on Trip Distance and Trip Rate

As was shown in the above section, tripmakers tend to trade off increases in speed for longer travel distances. Within the longer trip distances, they have a choice of making longer trips at the previous trip rate, more trips at the previous trip distance, more trips at shorter trip distance, or a combination of these alternatives in varying proportions.

The available data from Washington, DC and Twin Cities suggest that tripmakers prefer to increase their trip distances over their trip rates. Namely, they prefer to increase their spatial opportunities over trip purposes. The data from the former city show that the elasticity of trip distance vs speed during 1955-1968 was about 0.9, while that for the trip rate was about 0.1. In the latter city, however, the elasticities during 1958-1970 were about 0.7 and 0.3 respectively. Further analysis indicated that if the city has a relatively high population density, there is a higher incentive to increase trip distances, with a resulting strong dispersion of population. If, however, the city is already relatively dispersed, then increases in travel speeds will result in relatively lower increases in trip distances and higher increases in trip rates.

Here, then, is an interesting way of defining the benefits that may be derived from an improved transportation system: the induced travel can be divided into two principal components; of increased spatial opportunities, such as a wider choice of home-work O-D pairs, or of increased activities, such as more trips to social-recreation purposes. Hence, it will depend much on the preferences of the city authorities in what direction to steer the benefits; if more spatial opportunities are desired, then the city should be allowed to expand, but if more interaction of activities are preferred, then the expansion of the city should be restrained by land use control.

As it so often happens, both objectives may be desired, although in different proportions at different parts of the city. Hence, a clear definition of the desired benefits derived from an improved transportation system should first be stated before transportation planning can start. Furthermore, it becomes evident that such transportation planning may then be used as a lever to direct urban development in the desired direction, at least in cities that can control the uses of their land.
2.6 Travel Time Budgets in Cities of Developing Countries

As yet, there are no available data on the daily travel time of trip-makers in cities of developing countries. However, if the daily travel time per car is regarded as an indicator for the travel time of car trip-makers, it may be inferred that the TT-budget in developing countries is much higher than in developed countries. Figure 2.3 presents the daily travel time per car in a wide range of cities, and it becomes apparent that it is about 1.2 vs 0.8 hours in cities of developing countries vs developed countries respectively (7).

![Figure 2.3: Car Daily Traveltime vs. Motorization](image)

Although it is not yet known whether the higher car TT-budget in the former cities is due to a higher TT-budget per trip-maker or to more than one driver using the car during the day, the data do indicate that travel intensity in such cities is higher than in cities of developed countries (e.g. the car daily trip rate, as shown in Figure 3.10 of Ref. 7).

While this subject will be discussed in more detail later on, it can be mentioned here that two major factors seem to contribute to the higher travel intensities found in cities of developing countries: (i) the relatively high rates of economic development and growth requires higher rates of interaction and mobility, especially when the electronic system of communication (such as telephones) is underdeveloped; and (ii) some cities grow rapidly under the external pressure of migration, and urban structure does not have enough time to adjust itself to an equilibrium condition, thus forcing the inhabitants to travel more than expected in a relatively stable city.

The above indications suggest, therefore, that the two travel budgets of time and money are likely to be higher in the former cities. The time budgets, at least for cars, do indeed seem to be higher, and it will be shown later on that so also seem to be the money budgets.
2.7 In Conclusion

There are enough indications to suggest that tripmakers are constrained in their travel by two principal budgets -- of money and of time. It can then be assumed that tripmakers will tend to adjust their daily travel in such a way as to bring the two budgets into equilibrium.

One possible way to express this equilibrium condition is suggested in the following chapter.
3. Equilibrium Conditions of Travel

3.1 Introduction

The first stage of building the UMOT model, in its macro form, will be developed upon empirical relationships, simple in form and concept, based on static conditions but with dynamic ingredients. Each step of development will be checked for its consistency with various travel phenomena and characteristics, in order to ensure its integration within the general framework. Only at a later stage in this report will the UMOT model be assessed for its theoretical integrity, as detailed in Appendix 6. It should be noted that since most of the available data are based on the household unit, the following analyses are also based on the household characteristics, although at a later stage attention will be shifted to the tripmaker.

3.2 Equilibrium Between the Two Travel Budgets

The household's out-of-pocket expenditure on travel by a given mode is given by the product of distance travelled and the cost per unit distance, namely:

$$\text{Exp.} = D \cdot c$$  \hspace{1cm} (3.1)

If the household's daily travel time is $T$ (the product of the number of tripmakers per household and their TT-budget), it then follows that the value of the household's unit time of travel is:

$$\frac{\text{Exp.}}{T} = D \cdot c = v \cdot c$$  \hspace{1cm} (3.2)

where $v$ is the speed of travel.

The above relationship looks simple, but only deceptively so. For instance, it may be interpreted in the following way:

(1) The left-hand side is the ratio between the two principal travel budgets, which is strongly related to income and, therefore, represents travel demand by income;

(2) The right-hand side represents the two measures of the product that a transportation system supplies, namely speed of travel at a certain cost per unit distance;

(3) Hence, households at different income levels will tend to purchase from the transportation system different amounts of travel that will be proportional to the ratio between the household's two specific travel budgets;

(4) The equilibrium condition of travel in the urban area will be reached when the system supply will satisfy the travel demand of all households. Appendix 6 details some of the requirements for such an equilibrium condition. (As will be shown later, although travel in a city is regarded as a closed system on a daily basis, it is far from being in an equilibrium condition, and it is the forces that result from the non-equilibrium conditions which change and reshape the urban structure.) At this stage, a better insight into each one of the parameters in Eq. 3.2 is required, as detailed in the following sections.
3.3 The Cost Per Unit Distance of Travel in the US 1968

The analysis in this section will deal specifically with car travel, and the effect of speed on the cost per unit distance (henceforth, unit cost, for short). Transit costs will be discussed at a later stage.

In most analyses, the unit cost of car travel is divided into two major components: (i) operating costs, which are only incurred during travel, such as gas and oil consumption, tires, repairs and maintenance, as well as travel charges such as parking or tolls; (ii) standing costs, such as depreciation, registration and insurance, which are incurred even if the car is not in use.

While the operating costs are closely related to distance travelled, the standing costs are regarded as static. In order to combine them by a common denominator, both are transferred to costs per unit distance travelled. However, while the operating costs are sensitive to the speed of travel, the standing costs are allocated to the yearly average distance travelled by a car. This procedure does not consider the new empirical phenomenon, namely that the travel time per average car tends to be stable under all traffic conditions and, therefore, that the daily travel distance will change with speed. In other words, it now becomes apparent that all the standing costs have to vary with speed, as do the operating costs.

Figure 3.1, based on the detailed analysis in Appendix 1, shows how the unit cost of car travel changes with average travel speed. The unit costs can be related to the speeds, within the range of speeds found in cities, by the relationship

\[ c = 1.683 \cdot v^{-0.750} \]  

(3.3)

Referring back to Eq. 3.2, it can be seen that the right-hand side of the relationship, namely the product \( v \cdot c \), can now be expressed in a quantified way in relation to speed, as shown in Figure 3.2.

It may, therefore, be inferred that if the left-hand side of Eq. 3.2, the part that represents travel demand, will increase with income, then households at higher income levels will seek higher travel speeds in order to satisfy their amount of travel demand, as expressed by their two travel budgets. Therefore, the attention will now be shifted back to the left-hand side of Eq. 3.2.

---

1/ In some cases, depreciation is divided into two components; a part which is static and a part which varies with car usage.
Figure 3.1: Cost per Car-Mile vs. Speed, U.S., 1968 (Standard Size)

Figure 3.2: The "Measure of Travel" (v.c) vs. Speed, U.S., 1968
3.4 The Travel Cost Budget in Washington, DC 1968

Appendix 2 details, by district, the income, travel time and travel speed of households that made all their travel by car in Washington, DC 1968. The proportion of income allocated to car travel at different income levels, is given by the following relationship, based on Eqs. 3.2-3.3:

\[
\lambda = \frac{0.25}{1.683 \cdot \frac{v}{T} \cdot \frac{\text{Inc.}}{\text{Inc.}}}
\]  \quad (3.4)

where: \( \lambda \) - proportion of the household daily income allocated to travel;
\( v \) - weighted daily speed;
\( T \) - daily traveltime per household;
\( \text{Inc.} \) - the household average daily income.

The results of the analysis are detailed in Table 3.1 and shown in a graphical form in Figure 3.3. It becomes evident that the expenditure on car travel increases linearly with income, as already shown for travel conditions in the UK and London by all modes. Nonetheless, there is one important difference of car travel vs. travel by all modes: percentage-wise, expenditures on car travel tend to be relatively stable at all income levels.

Table 3.1: Daily Expenditure on Travel by Households making all Their Trips by Car, Washington, D.C., 1968

<table>
<thead>
<tr>
<th>HH Annual Inc. $</th>
<th>HH Daily Inc.</th>
<th>HH Daily Driver T, hr.</th>
<th>HH Daily Speed, mph.</th>
<th>Exp. as Proportion of Income</th>
<th>HH Daily Exp., $</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>19.23</td>
<td>0.67</td>
<td>11.7</td>
<td>0.108</td>
<td>2.08</td>
</tr>
<tr>
<td>7,000</td>
<td>22.44</td>
<td>0.77</td>
<td>16.3</td>
<td>0.116</td>
<td>2.60</td>
</tr>
<tr>
<td>8,000</td>
<td>25.64</td>
<td>0.88</td>
<td>19.8</td>
<td>0.122</td>
<td>3.13</td>
</tr>
<tr>
<td>9,000</td>
<td>28.85</td>
<td>0.98</td>
<td>22.5</td>
<td>0.124</td>
<td>3.58</td>
</tr>
<tr>
<td>10,000</td>
<td>32.05</td>
<td>1.08</td>
<td>24.8</td>
<td>0.126</td>
<td>4.04</td>
</tr>
<tr>
<td>11,000</td>
<td>35.26</td>
<td>1.18</td>
<td>26.7</td>
<td>0.128</td>
<td>4.51</td>
</tr>
</tbody>
</table>

When applying the proportions of income allocated to travel to the range of incomes, it becomes possible to define the left hand side of Eq. 3.2, representing travel demand, in quantified terms. Furthermore, these values would then represent the product \( v \cdot c \), namely a unique value for each income level, that represents system supply, as shown in Figure 3.2.

It may, therefore, be concluded that a household at a certain income level will be able to satisfy both travel budgets, of money and time, by purchasing from the transportation system a uniquely defined "measure of travel", and travel demand will reach an equilibrium condition if, and only if the transportation system will be able to supply this measure. And the "measure of travel" is expressed by speed and unit cost of travel.

As will be shown in the following sections, households owning a car are more likely to reach the equilibrium condition between their two travel budgets than households not owning a car.
3.5 Travel Equilibrium in Washington, D.C., 1968

The application of the equilibrium equation to the 1968 conditions in Washington, D.C. is given in great detail in App. 3; in order not to distract the attention from the main subject of the UMOT model, only the relevant results are presented in this section. (The calculation is done as an exercise only, and not as a proof for the UMOT model. The same applies to the exercise for Kuala Lumpur, described below).

(1) Equilibrium Between the Two Travel Budgets

While applying the equilibrium equation to households owning and not owning a car at different income levels, it becomes evident that households owning a car can reach equilibrium conditions at speeds that are within the range of speeds supplied by the transportation system. Households not owning a car, that have to rely on the slow transit travel only, reach the limit of their travel time budget much before their travel money budget has been expended, thus leaving them with a strong incentive to allocate the extra money for the purchase of higher speeds, such as those possible by car travel. Hence, the probability of a household owning a car will tend to increase with income when incomes cross the cost threshold of owning a car.

Table 3.2 summarizes the travel cost and time budgets for households owning and not owning a car at different income levels.
Table 3.2: Travel Cost and Time Budgets per Household, Washington, D.C., 1968

<table>
<thead>
<tr>
<th>HH Annual Income, $</th>
<th>6,000</th>
<th>7,000</th>
<th>8,000</th>
<th>9,000</th>
<th>10,000</th>
<th>11,000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Car Owners</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Budget, $</td>
<td>2.08</td>
<td>2.60</td>
<td>3.13</td>
<td>3.58</td>
<td>4.04</td>
<td>4.51</td>
</tr>
<tr>
<td>Time Budget, hr.</td>
<td>0.92</td>
<td>1.06</td>
<td>1.20</td>
<td>1.34</td>
<td>1.48</td>
<td>1.62</td>
</tr>
<tr>
<td>Driver Time, hr.</td>
<td>0.67</td>
<td>0.77</td>
<td>0.88</td>
<td>0.98</td>
<td>1.08</td>
<td>1.18</td>
</tr>
<tr>
<td>Speed, mph.</td>
<td>12.0</td>
<td>17.5</td>
<td>21.5</td>
<td>23.5</td>
<td>25.5</td>
<td>27.5</td>
</tr>
<tr>
<td>Exp. as % Inc.</td>
<td>10.8</td>
<td>11.6</td>
<td>12.2</td>
<td>12.4</td>
<td>12.6</td>
<td>12.8</td>
</tr>
<tr>
<td><strong>Non-Car Owners</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Budget, $</td>
<td>0.40</td>
<td>0.68</td>
<td>0.95</td>
<td>1.16</td>
<td>1.39</td>
<td>1.64</td>
</tr>
<tr>
<td>Time Budget, hr.</td>
<td>0.92</td>
<td>1.06</td>
<td>1.20</td>
<td>1.34</td>
<td>1.48</td>
<td>1.62</td>
</tr>
<tr>
<td>Speed, mph.</td>
<td>6.0</td>
<td>8.8</td>
<td>10.8</td>
<td>11.8</td>
<td>12.8</td>
<td>13.8</td>
</tr>
<tr>
<td>Exp. as % Inc.</td>
<td>2.1</td>
<td>3.0</td>
<td>3.7</td>
<td>4.0</td>
<td>4.3</td>
<td>4.7</td>
</tr>
<tr>
<td><strong>Weighted Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost Budget, $</td>
<td>1.01</td>
<td>1.85</td>
<td>2.67</td>
<td>3.36</td>
<td>3.94</td>
<td>4.45</td>
</tr>
<tr>
<td>Time Budget, hr.</td>
<td>0.92</td>
<td>1.06</td>
<td>1.20</td>
<td>1.34</td>
<td>1.48</td>
<td>1.62</td>
</tr>
<tr>
<td>Speed, mph.</td>
<td>7.9</td>
<td>13.6</td>
<td>18.8</td>
<td>22.2</td>
<td>24.9</td>
<td>27.1</td>
</tr>
<tr>
<td>Exp. as % Inc.</td>
<td>5.3</td>
<td>8.2</td>
<td>10.4</td>
<td>11.7</td>
<td>12.3</td>
<td>12.6</td>
</tr>
</tbody>
</table>

One surprising result is that the low proportions of expenditures on travel at low income levels, percentage-wise, as shown in Figures 1.1, 1.2, and 1.3, are the result of non-equilibrium conditions between the two travel budgets for non-car owning households, rather than an intrinsic characteristic of these households. Hence, the proportion of expenditure on travel to income increases rapidly with income — proportionally with the increase in motorization — towards a saturation level that would satisfy the equilibrium condition between the travel money and time budgets. As will be shown later on, the straining of the transportation system (in its most comprehensive aspect, including both travel demand and system supply) to reach equilibrium, is the mechanism which produces most of the forces that change and shape urban structure.

(2) Estimation of travel by the UMOT Model

Table 3.3 and Figure 3.4 present the estimated travel distance per household by income, both for travel by all modes and for car travel only, versus the observed travel distance, stratified by districts in Washington, D.C., 1968.

Table 3.3: Household Daily Travel Distance by Income, Washington, D.C., 1968 (Passenger-Miles)

<table>
<thead>
<tr>
<th>HH Annual Income Inc. $</th>
<th>HH Traveling by Car Only Estimated</th>
<th>HH Traveling by Car Only Observed</th>
<th>All Households Estimated</th>
<th>All Households Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>11.0</td>
<td>11.1</td>
<td>8.3</td>
<td>10.0</td>
</tr>
<tr>
<td>7,000</td>
<td>18.5</td>
<td>17.6</td>
<td>15.7</td>
<td>16.9</td>
</tr>
<tr>
<td>8,000</td>
<td>25.8</td>
<td>24.1</td>
<td>23.7</td>
<td>23.8</td>
</tr>
<tr>
<td>9,000</td>
<td>31.5</td>
<td>30.6</td>
<td>30.4</td>
<td>30.7</td>
</tr>
<tr>
<td>10,000</td>
<td>37.7</td>
<td>37.1</td>
<td>37.1</td>
<td>37.6</td>
</tr>
<tr>
<td>11,000</td>
<td>44.5</td>
<td>43.6</td>
<td>44.2</td>
<td>44.5</td>
</tr>
</tbody>
</table>
As can be seen, the similarity between the estimated and the observed values is very strong.

An additional test was to apply the UMOT model down to the level of individual districts, both for car and for total travel, as detailed in Appendix I and shown graphically in Figure 3.5.

It becomes evident that the UMOT model, even at its present macro-structure can estimate the passenger miles of travel at the district level with satisfactory results. The comparison of the total estimated vs. observed values are +2.7 percent for car travel only, and -3.4 percent for travel by all modes.
Figure 3.5: Estimated vs. Observed Person-Miles of Travel, by District, Washington, D.C., 1968

(3) Estimation of Modal Splits by Distance and by Trips

An additional test was to compare modal split ratios, as derived from the UMOT model, with the observed values. It should, however, be noted that the observed modal splits, as expressed in Figure 3.6, are based on the ratios between the person trips done by households who made all their trips by transit only vs. house-
holds who made all their trips by car only while the estimated modal splits are for all person trips. Nonetheless, it may be assumed that the observed 'net' modal splits represent the proportions of the 'gross' modal splits. (1)

Figure 3.6 shows, then, the comparison between the estimated and the observed modal splits, where it becomes apparent once again that the UMOT model, based on the two travel budgets, can express the observed modal splits fairly accurately, both for trips and for distance travelled.

Figure 3.6: Modal Split by Travel Distance and by Person Trips vs. Household Motorization, Washington, D.C., 1968

(1) Trips made by households who used mixed modes were not differentiated by mode in the available summary tables and, therefore, could not be used for modal split calculations.
3.6 The Effect of Travel Equilibrium on Residence Location

The results up to now indicate that tripmakers at increasing income levels will tend to purchase increasing travel speeds, and travel longer distances, within the equilibrium conditions of their travel money and time budgets. Insofar as travel patterns are distributed in urban areas, the general characteristic is that travel speeds increase with distance from the center. It may, therefore, be inferred that tripmakers will tend to change their locations and travel patterns in such a way as to increase their daily total average speed; namely, they will tend to disperse their residence location from the city's center, and prefer to travel to destinations that can be reached at high speeds. Thus, in a city which is allowed to expand freely, the interaction between travel demand, system supply and urban structure can be visualized by the following process:

(i) Increases in income will generate a demand for higher travel speeds, thus encouraging the increase in motorization;

(ii) If the transportation system will be developed to allow increased speeds, than the tripmakers (or, rather, their households) will tend to disperse from the city center, in step with the equilibrium conditions of their two travel budgets and, at the same time, fulfilling many of their varied desires, such as residence size, neighborhood, and schools for the children.

It should be pointed out at this stage that the equilibrium condition between the travel money and time budgets is considered in this report not to be the cause for, nor the effect of some law of nature, by which tripmakers must abide, but only as a component - though a major one - in a more complex mechanism of human behavior under urban conditions. Hence, many additional factors have to be considered as well, such as rent, availability of utilities, household size, age and availability of a driving license, in order to be able to predict in more detail the behavior of populations within the urban areas. Nonetheless, the advantage of the equilibrium condition is in its ability to bring together, under one mechanism, travel demand, system supply and urban structure, as well as explain many seemingly unrelated travel and urban phenomena in a unified and consistent way. As an example, the distribution of households by income in the Washington, D.C. area in 1966 can be explained in the following way: (see App. 3 for the supporting data).

(1) The distance of households from the city's center increases with income according to the relationship:

\[ d = 0.00177 \text{ Inc.} - 10.13 \quad (r = 0.906); \quad (3.5); \]

where: \( d \) = distance from center, miles; 
\( \text{Inc.} \) = household annual income, Dollars

This relationship is shown in Table 3.4 and Figure 3.6 for a range of incomes;

(2) The daily average speed of travel by households increases with distance from the center according to the relationship:

\[ d = 0.538 \text{ v} -7.51 \quad (r = 0.843); \quad (3.6); \]
(3) The speeds of car owning households, as derived from the UMOT model for a range of incomes (detailed in App. 3 and summarized in Table 3.1), were substituted in Eq. 3.6 in order to derive the estimated distances of the households' residence from the center. The results are plotted in Figure 3.7 for the range of incomes.

<table>
<thead>
<tr>
<th>HH Annual Income, $</th>
<th>HH Exp. $</th>
<th>Driver T, hr.</th>
<th>( v_c = \frac{\text{Exp.}}{T} )</th>
<th>Speed mph</th>
<th>Distance from Center, m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>2.08</td>
<td>0.67</td>
<td>3.10</td>
<td>12.0</td>
<td>(-)</td>
</tr>
<tr>
<td>7,000</td>
<td>2.60</td>
<td>0.77</td>
<td>3.38</td>
<td>17.5</td>
<td>1.9</td>
</tr>
<tr>
<td>8,000</td>
<td>3.13</td>
<td>0.88</td>
<td>3.56</td>
<td>21.5</td>
<td>4.1</td>
</tr>
<tr>
<td>9,000</td>
<td>3.58</td>
<td>0.98</td>
<td>3.65</td>
<td>23.5</td>
<td>5.1</td>
</tr>
<tr>
<td>10,000</td>
<td>4.04</td>
<td>1.08</td>
<td>3.74</td>
<td>25.5</td>
<td>6.2</td>
</tr>
<tr>
<td>11,000</td>
<td>4.51</td>
<td>1.18</td>
<td>3.82</td>
<td>27.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

(1) Distance from Center = 0.538 Speed - 7.51 (\( r = 0.843 \))

**Figure 3.7**: Estimated vs. Observed Distance of Households from the City Center, by Income, Washington, D.C., 1968

It becomes evident from Figure 3.7 that the households owning a car, at various income levels, are expected to reside at specific distances from the center (distances that would be required in order to bring into equilibrium the households' two travel budgets) which are similar to the distances observed.

The above example (with no calibration attempted) indicates that the equilibrium condition of the travel money and time budgets may be used as a boundary condition for the spatial location of households in the urban area. Namely, it would indicate the optimal location, without ensuring a-priori that the households would indeed reach their equilibrium condition of travel and spatial location. This subject will be further elaborated in chapter 5 when discussing urban structure under various constraints.
3.7 Elasticity of Travel Distance vs. Travel Cost

It has already been shown above that the various travel parameters interact with each other in different proportions, resulting in different relationships. For instance, travel expenditures tend to increase with income, resulting in increases in speed and travel distance, while the unit cost of travel tends to decrease with increases in speed within the range observed in cities. One of the intriguing results is that when speeds are increased by improving the system supply, it then follows that if households at a given income level are to take advantage of the increased speed, in order to travel farther within their TT-budget, (as they are observed to do), then their travel cost budget must increase as well. How, then can the increasing proportions of travel cost budgets to income under such conditions, for fixed income levels, be reconciled with the indication that such proportions tend to be stable? One possibility is to assume that since incomes increase with time, they can explain the increase in expenditures and distance travelled, without having to rely on the suggested mechanism of travel. However, increases of income with time cannot yet explain wide variations of travel behavior during the same time period. Such a case is shown in Table 3.5 for Washington, D.C. (1968), Twin Cities (1970) and all the U.S. (1970), where it becomes evident that the travel during the same year in Twin Cities (urban travel only) and all the U.S. (urban and inter urban travel) are widely different distance-wise, although they are practically the same time-wise. In other words, the increase in travel distances (and expenditures) due to increase in speed cannot be explained solely by the increase of incomes with time.

Table 3.5: Speed Trade-off for More Travel Distance, Car Tripmakers

<table>
<thead>
<tr>
<th>Study Area</th>
<th>Year</th>
<th>T, hr.</th>
<th>Speed, kph</th>
<th>Increase in Speed</th>
<th>Daily Distance, km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washington, D.C.</td>
<td>1955</td>
<td>1.09</td>
<td>18.8</td>
<td>+23.9</td>
<td>20.5</td>
</tr>
<tr>
<td></td>
<td>1968</td>
<td>1.11</td>
<td>23.3</td>
<td></td>
<td>25.9</td>
</tr>
<tr>
<td>Twin Cities</td>
<td>1958</td>
<td>1.14</td>
<td>21.4</td>
<td>+32.9</td>
<td>24.5</td>
</tr>
<tr>
<td></td>
<td>1970</td>
<td>1.13</td>
<td>28.5</td>
<td></td>
<td>32.3</td>
</tr>
<tr>
<td>All U.S.</td>
<td>1970</td>
<td>1.10</td>
<td>45.6</td>
<td></td>
<td>50.3</td>
</tr>
</tbody>
</table>

In the search for a possible answer to the above question, it became evident that due to the relationship between the unit cost of travel and speed, the total cost increases with income at decreasing rates. This relationship is detailed in Table 3.6 and Figure 3.8, where it can be seen that the elasticity of travel distance vs. travel cost for car travel is much above unity.

The interpretation of this result is that an increase in speed is a strong incentive for tripmakers to increase their travel distance - and spatial opportunities - at only marginal increases in cost. Furthermore, since spatial opportunities(1) are considered as synonyms with greater income opportunities, it may be inferred that tripmakers will be willing to increase

(1) Sometimes expressed by accessibility measures, such as the number of jobs that can be reached within 45 minutes of travel.
Table 3.6: Elasticities of Travel Distance vs. Travel Cost, Car Tripmakers, Washington, D.C., 1968

<table>
<thead>
<tr>
<th>HH Annual Income, $</th>
<th>Daily Exp. $</th>
<th>Daily Distance Miles</th>
<th>Elasticity (1) Dist./Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,000</td>
<td>2.08</td>
<td>11.0</td>
<td>2.29</td>
</tr>
<tr>
<td>7,000</td>
<td>2.60</td>
<td>18.5</td>
<td>1.78</td>
</tr>
<tr>
<td>8,000</td>
<td>3.13</td>
<td>25.8</td>
<td>1.49</td>
</tr>
<tr>
<td>9,000</td>
<td>3.58</td>
<td>31.5</td>
<td>1.48</td>
</tr>
<tr>
<td>10,000</td>
<td>4.04</td>
<td>37.7</td>
<td>1.51</td>
</tr>
<tr>
<td>11,000</td>
<td>4.51</td>
<td>44.5</td>
<td>1.17</td>
</tr>
<tr>
<td>12,000</td>
<td>5.00</td>
<td>50.2</td>
<td></td>
</tr>
</tbody>
</table>

(1) Mid-point elasticities

Figure 3.8: Elasticity of Travel Distance by Car vs. Cost of Travel, Washington, D.C., 1968

their travel cost above their previous allocation if they will expect it to increase their benefits, including an increase in their income, until the transportation market will stabilize itself at approximately the same proportions of expenditure to income as before. This indication is corroborated by the established finding that a better transportation system often results in a higher and better economic activity, increasing the utility of its users. Thus, the indicated increase in expenditure by an increase in speed does not violate but rather corroborates, the observed relationships.

When considering the effect of increased speed on transit travel, the situation becomes less defined, since transit fares may be based on different measures, such as per unit distance, per unit distance plus speed (express service), or a flat fare, either by zones or for the total city. Nonetheless, the equilibrium equation does indicate one important aspect of transit usage, at least in cities of developed countries: the money budget is not the critical factor, even at low income levels, since the time budget becomes exhausted much before the money budget does. This result may explain not only why transit travel is inelastic to price but also
why shared taxis are such a success, although they are more expensive than buses. But the best illustration for the effect of speed on transit travel is probably the case where a rapid transit is in operation on its own right-of-way, independent of the road network speeds. Table 3.7 details such an example, for London, at two points in time.

Table 3.7: Change of Person Trips Over Time, London 1962-1971

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>1962</th>
<th>1971</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>8,857,000</td>
<td>8,372,000</td>
</tr>
<tr>
<td>Cars</td>
<td>1,249,450</td>
<td>1,885,700</td>
</tr>
<tr>
<td>Motorization, %</td>
<td>14.1</td>
<td>22.5</td>
</tr>
<tr>
<td>Car Person Trips</td>
<td>5,411,000</td>
<td>7,377,000</td>
</tr>
<tr>
<td>Road Transit Person Trips</td>
<td>4,039,330</td>
<td>3,094,330</td>
</tr>
<tr>
<td>Rapid Transit</td>
<td>1,834,000</td>
<td>2,025,000</td>
</tr>
</tbody>
</table>

As can be seen, while population remained relatively stable during 9 years, motorization increased by 60 percent. As a result, the car travel increased by nearly 2 million person trips while the road transit (buses + taxis) lost nearly 1 million person trips. Nonetheless, the number of person trips in the rapid transit system did not decrease, as expected, but even increased slightly from 1.8 million to over 2 million. It may, therefore, be concluded that transit travel is more sensitive to speed than to cost, at least in cities of developed countries, although the success of shared taxis in cities of developing countries indicates that the same conclusion applies there as well.

3.8 Travel Equilibrium Under Policy Changes

The equation of travel equilibrium may also be assessed under exogenous changes, such as those brought by policy decisions. Such examples are discussed below:

1. Change in unit costs.
   If the unit costs of travel are increased, as happened in 1973/74, tripmakers will have to economize on their total distance travelled. Eq. 3.2, namely, Exp./T = v.c, indicates that in order to keep the left hand side stable, speed will have to be reduced (such as by the relocation of residence towards the city's center). Inevitably, therefore, the total distance, within the TT-budget, will tend to decrease. However, since tripmakers would be unwilling to decrease their daily travel, most of the decrease in cost would have to be funnelled into other directions, such as economizing on travel during weekends, delay the change of the car to a new model, or change it to a smaller and more economical car. All these alternatives, to one degree or another, were actually observed. One thing, though, becomes evident from the equilibrium equation; not many car tripmakers are expected

1. Incidentally, the speed limits were, indeed, reduced simultaneously with increases in the gas cost in 1973/74, although for different reasons. Furthermore, the real estate market reacted to such changes by increasing land values at and near the center, although the expectations for mass population return towards the center did not materialize.
to divert to transit, since it would disrupt their travel equilibrium much more than by a relatively small change in the unit costs of car travel.

(2) **Free transit**

If transit were made free of charge, the expected result is that trips would tend to increase somewhat, by two factors:

(i) by tripmakers at very low income levels, who cannot afford even bus fares;

(ii) short walking trips would be diverted to transit if walking time and effort can be reduced.

But the most surprising indication, however, is that making transit free of charge would also increase the available money budgets allocated to travel, thus increasing the incentive to purchase a car by households who do not own one, or increase the amount of travel by car owning households who also use transit. Although there are no actual cases where transit was made free throughout an urban area and, therefore, the above indication cannot be verified by experience, the possible result cannot be disregarded. Furthermore, when transit fares were reduced drastically (Atlanta, Georgia), or when transit made free in the city center (Rome, Italy), the number of transit person trips increased mostly by the diversion of short walking trips, with no noticeable effect on the total car travel within the city.

As can be seen from the above examples, although the travel equilibrium equation is still simple in structure and application, it already can explain some intriguing observations that have been baffling transportation planners for some time, such as induced travel, inelasticity of transit travel to cost, resistance of car tripmakers to divert to transit, and the relative stability of rapid transit vs. road transit at increasing motorization levels.

The last example of the interpretation of the equilibrium equation is when considering it for tripmakers (or their households) at very high income levels, beyond the range of available data. It becomes apparent that the need for spatial opportunities would increase with income, and, therefore, the willingness to allocate more money for travel would increase as well. Although no data are available to indicate the shape of the relationship at very high incomes (i.e., linear or at decreasing rates), it becomes evident that the required travel speed will quickly reach and surpass the maximum speed that the road network can supply. It is reasonable to expect that such tripmakers would then shift to more rapid modes than the car, such as commercial aeroplane flights at medium-high incomes and privately owned aeroplanes at very high income levels. (These possibilities suggest that travel behavior on the basis of travel money and time budgets, even at low income levels, should be based on a weekly or, even better, on a monthly basis, than on a daily basis).

Another outlet for the travel money budget at high income levels, for other purposes then just trips to work, is expending on such modes as boats. Altogether, then, the increase in the travel money budget with income does not seem to stop with car travel, and some specific income thresholds between different travel modes become apparent.
The Effect of Travel Cost on Motorization Levels

The relationship between the travel cost expenditure and the income level suggests that there has to be a certain threshold of income below which households cannot afford to own a private car. Indeed, in the case of Washington, D.C., 1968, the district average threshold was about $7,000 per year.

It may further be inferred that the threshold will rise up the income scale if the cost of owning and operating a car increases. An example for such a case may be found in New York, where the travel cost by car within the city is much higher than in the suburbs (both operating and parking costs). As a result the motorization level within the city is much lower than in the suburbs, for the same income levels, as can be seen in Table 3.8 and Figure 3.9.

Table 3.9: Percent of Households Owning a Car, by Income Level, New York, 1963

<table>
<thead>
<tr>
<th>HH Annual Inc. $('000)</th>
<th>Households Owning a Car, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New York City</td>
</tr>
<tr>
<td>0 - 2</td>
<td>7</td>
</tr>
<tr>
<td>2 - 3</td>
<td>12</td>
</tr>
<tr>
<td>3 - 4</td>
<td>16</td>
</tr>
<tr>
<td>4 - 5</td>
<td>28</td>
</tr>
<tr>
<td>5 - 6</td>
<td>31</td>
</tr>
<tr>
<td>6 - 7.5</td>
<td>55</td>
</tr>
<tr>
<td>7.5-10</td>
<td>67</td>
</tr>
<tr>
<td>10 - 15</td>
<td>72</td>
</tr>
<tr>
<td>15 &amp; Over</td>
<td>73</td>
</tr>
</tbody>
</table>

Figure 3.9: Percent of Households Owning a Car vs. Income, New York, 1963

The above results, therefore, suggest a method by which the long range effects of increases in the cost of travel by car, either by creeping congestion or by policy actions, may inhibit the increase in motorization. Such a method will also allow a better assessment of the possible long range effects of local road pricing policy on the motorization levels in the city and their resulting amounts and patterns of travel.
3.10 In Conclusion

Although the equation of travel equilibrium seems to be very simple and self-evident, it has some fundamental meanings and implications. It also lends itself, as detailed in Appendix 6, to straightforward maximization, within the double constraints of the two travel budgets, of either the total travel distance - as a direct measure of person miles of travel, or of the total area covered by travel - as a direct measure of spatial opportunities. These results, therefore, also suggest new techniques for direct distribution and assignment of travel in the urban area, by travel fields rather than by single trips.
4. Travel in Cities of Developing Countries

4.1 Introduction

Most of the previous examples and discussions referred to cities in developed countries, which are characterized by expansion from within, by a process of differential growth. Thus tripmakers residing in the area have some freedom to adjust their location and choice of O-D patterns in line with their travel equilibrium condition. In other words, they satisfy their travel need by dispersion.

In cities of developing countries, on the other hand, expansion is very often from the outside, by the migration of households and individuals from the poor hinterland to the relatively rich city. Hence, such a city tends to expand by the migration of poor newcomers, who increase the internal pressure within the city by settling in its outskirts. Furthermore, the inadequate transportation system, both the road and transit networks, becomes rapidly overloaded by the ever-increasing pressure of travel demand. If one adds to the above scenario a doubling of population every decade, coupled with a rapid increase in motorization by the relatively rich inhabitants within the central city, the severity of the transportation situation may be appreciated.

At first, it would seem that the above scenario would conflict with the principles of the UMOT model, since such cities would be far from a condition of travel equilibrium. This chapter, therefore, will be devoted to the analysis of travel by the UMOT model in a city from the developing countries—Kuala Lumpur. The purpose of the analysis is to show that the basic mechanism of urban travel applies to developing countries also, despite significant differences in some parameters.

4.2 Unit Cost of Travel in Kuala Lumpur

The first step of analysis is to review the unit costs of travel found for Kuala Lumpur (henceforth, K.L.) and compare them with those in the U.S. (6 vs 8 and 2). In the consultants' studies preceding the Second Kuala Lumpur Urban Transport Project the costs of fuel, lubricants and tires were found to be M\$ 7.3 per mile, and the tax on them M\$ 3.8 per mile. Capital charges (based on a vehicle cost of M\$ 8,000 and an 8-year life) were M\$ 2,000 pa. and taxes on car purchase and ownership M\$ 150 pa. The observed daily average mileage per car was found to be 6.75 trips x 3.35 miles equals 22.7 miles, equivalent to a total mileage of 8,300 in a 365-day year. Under these conditions, average unit costs in KL in 1973 were about M\$ 0.11 for a car-mile.

Referring back to Appendix 1, and in Figure 4.1, it becomes evident that the unit cost in the U.S. (1972) for a subcompact car, based on 16.3 mph (the daily average agreed in K.L.) is about US\$ 15 which, based on an exchange rate of US\$ 1 = M\$ 2.5, is equivalent to 37.5 M\$. The K.L. value of US\$ 0.11 M\$ per car-mile is consistent with the U.S. value.
4.3 Private Vehicle-miles of Travel K. L. 1972

The estimation of the car and motorcycle daily miles of travel is summarized in Table 4.1, based on the following data and relationships:

1. The number of tripmakers per household = 0.477 Motorization + 1.509 (Ref. 7, Figure 3.11);
2. The observed car daily TT-budget, of 1.4 hours, applied to all private vehicle drivers;
3. The speed of travel, as derived from the alpha-relationship of K.L. (10);
4. The number of households grouped by availability of private vehicle - as observed (5).

Table 4.1: Private Vehicle-Miles of Travel, K.L. 1972

<table>
<thead>
<tr>
<th>Car Availability</th>
<th>2 Cars</th>
<th>1 Car</th>
<th>Motorcycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tripmakers/HH</td>
<td>2.46</td>
<td>1.99</td>
<td>1.99</td>
</tr>
<tr>
<td>Drivers/HH</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Daily TT-budget, hr.</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Driver Traveltime, hr.</td>
<td>2.8</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>Speed, MPH</td>
<td>16.3</td>
<td>16.3</td>
<td>16.3</td>
</tr>
<tr>
<td>Daily Distance/HH</td>
<td>45.6</td>
<td>22.8</td>
<td>22.8</td>
</tr>
<tr>
<td>Households</td>
<td>10,845</td>
<td>41,417</td>
<td>30,751</td>
</tr>
<tr>
<td>Vehicle Miles of Travel</td>
<td>494,970</td>
<td>945,140</td>
<td>701,740</td>
</tr>
<tr>
<td>Total Estimated VMT</td>
<td>1,440,110</td>
<td></td>
<td>701,740</td>
</tr>
<tr>
<td>Total Observed VMT</td>
<td>1,486,225</td>
<td></td>
<td>683,730</td>
</tr>
<tr>
<td>Diff. Est./Obs., %</td>
<td>-3.1</td>
<td></td>
<td>+2.6</td>
</tr>
<tr>
<td>Total Estimated</td>
<td>2,141,850</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Observed</td>
<td>2,169,955</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. Est./Obs., %</td>
<td>-1.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
As can be seen in Table 4.1, the estimated private vehicle-miles of travel (VMT) is 2,141,850, while the observed value is 2,170,000, namely a difference of -1.3 percent.

4.4 Person-miles of Travel by Mode

Table 4.2 details the summary of the estimated person miles of travel by mode, based on the number of tripmakers per household and their TT-budgets, while the detailed calculations are given in Appendix 5.

Once again, the estimated vs. observed values are very similar, with a difference of only -2.6 percent.

Table 4.2: Person Miles of Travel, K.L., 1972

<table>
<thead>
<tr>
<th>Car Availability</th>
<th>2 Cars</th>
<th>1 Car</th>
<th>M/c</th>
<th>Non (Transit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM/HH - Private</td>
<td>2.46</td>
<td>1.84</td>
<td>1.15</td>
<td>-</td>
</tr>
<tr>
<td>TM/HH - Transit</td>
<td>-</td>
<td>0.15</td>
<td>0.84</td>
<td>1.51</td>
</tr>
<tr>
<td>TM/HH - Total</td>
<td>2.46</td>
<td>1.99</td>
<td>1.99</td>
<td>1.51</td>
</tr>
</tbody>
</table>

Private

<table>
<thead>
<tr>
<th>Estimated PMT</th>
<th>608,080</th>
<th>1,741,580</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed PMT</td>
<td>2,349,660</td>
<td>806,910</td>
</tr>
<tr>
<td>Diff. Est./Obs. %</td>
<td>-3.0</td>
<td>+0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Est. Total PMT</th>
<th>3,156,570</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs. Total PMT</td>
<td>3,222,510</td>
</tr>
<tr>
<td>Diff. Est./Obs. %</td>
<td>-2.1</td>
</tr>
</tbody>
</table>

Transit

<table>
<thead>
<tr>
<th>Estimated PMT</th>
<th>87,390</th>
<th>358,560</th>
<th>1,536,820</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Total</td>
<td>1,982,770</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed Total</td>
<td>2,056,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diff. Est./Obs. %</td>
<td>-3.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total

<table>
<thead>
<tr>
<th>Estimated Total PMT</th>
<th>5,139,340</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Total PMT</td>
<td>5,278,510</td>
</tr>
<tr>
<td>Diff. Est./Obs. %</td>
<td>-2.6</td>
</tr>
</tbody>
</table>

| Est. Modal Split, % | 38.6 |
| Obs. Modal Split, % | 38.9 |

4.5 Person Trips, by Mode

Table 4.3 presents the summary of the estimated person trips, by mode, based on the above person-miles of travel and the average trip distance. Following the previous results, the difference between the estimated and the observed total number of person trips is -2.5 percent.

Furthermore, even the modal split of estimated trips between the transit and the private modes is very similar to the observed one, namely 38.6% vs. 39.0% respectively.
Table 4.3: Person Trips, K.L., 1972

<table>
<thead>
<tr>
<th>Mode</th>
<th>Estimated</th>
<th>Observed</th>
<th>Diff. Est./Obs. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private</td>
<td>942,260</td>
<td>960,040</td>
<td>-1.9</td>
</tr>
<tr>
<td>Transit</td>
<td>591,870</td>
<td>613,730</td>
<td>-3.6</td>
</tr>
<tr>
<td>Total</td>
<td>1,534,130</td>
<td>1,573,770</td>
<td>-2.5</td>
</tr>
<tr>
<td>Modal Split %</td>
<td>38.6</td>
<td>39.0</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

### 4.6 Estimating the Travel Costs in K.L.

After verifying that the TT-budget alone can simulate travel conditions in K.L., it now becomes possible to apply the equation of travel equilibrium in order to derive estimates of the travel cost budgets in K.L. The steps of calculation are detailed in Appendix 5, while the results are summarized in Table 4.4.

Table 4.4: Travel Costs in K.L., 1972

<table>
<thead>
<tr>
<th>Car Availability</th>
<th>2 Car</th>
<th>1 Car</th>
<th>M/c</th>
<th>Non (Transit)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distance/HH, m.</strong></td>
<td>45.6</td>
<td>22.8</td>
<td>22.8</td>
<td></td>
</tr>
<tr>
<td><strong>Unit Cost, M$</strong></td>
<td>41.0</td>
<td>41.0</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td><strong>Cost/HH, M$$</strong></td>
<td>18.7</td>
<td>9.4</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td><strong>Distance/HH, m.</strong></td>
<td></td>
<td>2.1</td>
<td>11.7</td>
<td>20.9</td>
</tr>
<tr>
<td><strong>Unit Cost, M$</strong></td>
<td></td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td><strong>Cost/HH, M$$</strong></td>
<td></td>
<td>0.13</td>
<td>0.70</td>
<td>1.25</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Cost, M$$</strong></td>
<td>18.70</td>
<td>9.53</td>
<td>3.00</td>
<td>1.25</td>
</tr>
<tr>
<td><strong>Inc./Month, M$$</strong></td>
<td>1,890</td>
<td>856</td>
<td>416</td>
<td>338</td>
</tr>
<tr>
<td><strong>Exp./Inc. %</strong></td>
<td>25.7</td>
<td>28.9</td>
<td>18.7</td>
<td>9.6</td>
</tr>
</tbody>
</table>

As can be seen, applying unit costs of M$ 41 per car-mile and M$ 10.0 per motorcycle-mile results in relatively very high money expenditures on travel. The expenditures and their proportion by income are shown in Figure 4.2, where it can be seen that the high expenditures have already reached their saturation level at the income level of households owning 1 car, and they even start to decline somewhat, percentage-wise, for households owning 2 cars.

In concluding this exercise, it becomes evident that since cars - and presumably their tripmakers as well - in cities of developing countries travel for longer times during the day at lower speeds, than in cities of developed countries, the inevitable conclusion is that the expenditures on travel in the former cities, as a percent of the households' income, are much higher than in the latter cities, thus putting a heavy burden on the population of the rapidly developing cities.
The above results emphasise three important factors for the planning of transportation and urban form in these cities:

1. Travel demand in cities of developing countries, expressed by the travel money and time budgets, is much higher than in cities in developed countries.

2. The high travel costs in cities of developing countries, which put a heavy burden on household incomes and on tripmakers' time, suggest that the improvement of travel conditions in such cities deserves high priority.
(3) Of special concern is the urban structure in many of the developing cities, which expand by the peripheral settlement of poor newcomers, who have to travel the longest in search of work. Even Kuala Lumpur, which is relatively small and stable when compared with large cities, such as Bangkok, shows signs of very high travel time and money budgets, a sign that should call the attention of the urban planners to the unusually high travel demand in such cities.

Hence, the coordination of transportation and urban planning in cities of developing countries seems to be of even higher priority and importance than practiced today in cities of developed countries.

4.7 In Conclusion

The suggested mechanism of travel, as expressed by the travel equilibrium formulation, may be applied in cities of developing countries in many ways. First, and foremost, is its ability to bring together the measures of travel demand and system supply, and express the conditions under which equilibrium between the two may be achieved. Secondly, alternative urban and transportation plans may be evaluated in light of their effects on the two travel budgets, in order to bring them down to the levels observed in cities of developed countries. And thirdly, the UMOT model can also be used as a useful tool for urban planning, as such, including the spatial distributions of, and interactions between, population and land uses in the urban area. The last subject is further elaborated in the next chapter, where it is shown that one component of the complete UMOT model, the mechanism of differential accumulation of population vs. jobs, may serve not only as an independent tool for urban planning, but may also serve as a link between the urban structure and travel, thus shedding more light on the interaction between the two.
5. The Urban Structure

5.1 Introduction

Urban structure is complex, representing a compromise between a variety of interacting factors - geographic, climatic, historical, social and economic. Hence, it is impossible to describe all the nuances of urban structure by one process, and many different mechanisms are usually required, each one for a different urban activity.

This chapter will suggest a new process, that describes the interaction between the spatial distributions of population and jobs in an urban area, and which seems to serve as a link between the urban structure and the amount and patterns of travel within the city. The general description and application of the process is described in this chapter, while the mathematical interpretation of it is detailed in Appendix 6.

5.2 The Differential Accumulation Process

The first step is to describe the location of workers and jobs in the city by the distribution of their distances from the city center. (If no data on workers are available, then a surrogate such as households or population can be used as an approximation). In order to explain the process and ease the computations, the two distributions will be described by rings around the city center, and the distances will be from their midpoints to the center.

The differential accumulation is then defined as the accumulation of the differences between the two distributions towards the city center, in normalized values (assuming that the numbers of workers and jobs within the city are equal). Examples of differential accumulation curves are presented in Figure 5.1 for a wide selection of cities from both developing and developed countries, based on the detailed data in Appendix 7. It becomes evident that the differential accumulation (henceforth, D.A., for short) curves are similar in form, although they vary in shape and scale: they increase slowly towards the center, reaching a peak near the center and then fall sharply down to zero at the center itself.\(^1\)

The family of D.A. curves may be expressed by a Gamma distribution in the form of:

\[
F(x) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}
\]  

(5.1)

where:  
\(\alpha\) is the shape parameters; 
\(\beta\) is the scale parameter; 
\(x\) is the base of natural logarithms; 
\(\Gamma\) is the Gamma function of \(\alpha\), or \((\alpha-1)!\); 
\(F(x)\) is the relative density of occurrences.

\(^1\) The few cases where a second bump appears near the edge of the city represent either an expressway around the city, or regional characteristics.
The steps of fitting a Gamma function to observations by the maximum likelihood estimation technique was already detailed in a previous paper (2) while a more simplified technique is detailed in Appendix 6. The interpretation of the D.A. curves, and their possible applications, are presented in the following sections.

5.3 Estimating Average Trip Distances by the D.A. Function

Analysis of the mathematical properties of the D.A. function shows that the distance from the D.A. centroid to the city center can be approximated by:

\[ d = \frac{1}{2} (d_p + d_g) \]  \( (5.2) \)

where \( d_p \) is the average distance of workers' residences from the city center, and \( d_g \) is the average distance of job locations from the city center. The approximation can be improved by adding a correction term given by:

\[ \frac{\sigma_p^2 - \sigma_g^2}{2(d_p^2 - d_g^2)} \]  \( (5.3) \)

where \( \sigma_p \) and \( \sigma_g \) are the variances of the two distributions of the distance of workers' residences and jobs, respectively, from the city center.

While searching for the interpretation of \( d \) in observable terms in the city, it was found that the distance between the D.A. centroid and the city center corresponded very closely with the observed average trip distance in the city. Indeed, further mathematical analysis indicated that the average travel distance to work cannot be less than the absolute value of the difference between the average distance of the workers from the city center and the average distance of the jobs from the city center.

Hence, it may be inferred that the average trip distance to work is strongly dependent on the spatial distributions of households and jobs in the urban area and may, therefore, be derived directly from these distributions. Moreover, it also became apparent that the D.A. function can also approximate the weighted average trip distance, for all trip purposes, with satisfactory accuracy, as shown in Table 5.1.

Table 5.1: Estimated vs. Observed Car Average Trip Distance, km.

<table>
<thead>
<tr>
<th>City</th>
<th>Year</th>
<th>( \sum F \times X )</th>
<th>( \sum F )</th>
<th>( \bar{d} = \frac{\sum F \times X}{\sum F} )</th>
<th>( d ) Obs.</th>
<th>Diff. ( \bar{d} - \bar{d} ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens</td>
<td>1962</td>
<td>301.3</td>
<td>54.5</td>
<td>5.5</td>
<td>5.6</td>
<td>-1.8</td>
</tr>
<tr>
<td>Kuala Lumpur</td>
<td>1972</td>
<td>457.4</td>
<td>90.6</td>
<td>5.1</td>
<td>5.3</td>
<td>-3.8</td>
</tr>
<tr>
<td>Tel Aviv</td>
<td>1965</td>
<td>335.3</td>
<td>81.6</td>
<td>4.1</td>
<td>4.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Bangkok</td>
<td>1972</td>
<td>305.7</td>
<td>42.1</td>
<td>7.3</td>
<td>7.4</td>
<td>-1.4</td>
</tr>
<tr>
<td>Baltimore</td>
<td>1962</td>
<td>667.0</td>
<td>73.8</td>
<td>9.0</td>
<td>9.3</td>
<td>-3.2</td>
</tr>
<tr>
<td>Absolute Normalized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Washington, D.C.</td>
<td>1955</td>
<td>7,810,403</td>
<td>1,176,647</td>
<td>6.64</td>
<td>6.68</td>
<td>-0.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>1968</td>
<td>22,814,018</td>
<td>2,258,634</td>
<td>10.10</td>
<td>10.62(1)</td>
<td>-4.9</td>
</tr>
<tr>
<td>&quot;</td>
<td>1958</td>
<td>4,948,474</td>
<td>640,916</td>
<td>7.72(2)</td>
<td>(6.76)(3)</td>
<td>+14.2</td>
</tr>
<tr>
<td>&quot;</td>
<td>1970</td>
<td>6,049,555</td>
<td>754,767</td>
<td>8.02</td>
<td>8.40</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

The interpretations and applications of the above result are detailed in the next sections.

(2) Office Memorandum, "City Structure and Mobility", November 7, 1975.
5.4 The D.A. Function and City Structure

The results of both the empirical comparisons and the mathematical analyses of the properties of the D.A. function suggest the following process within the urban area:

(1) People tend to distribute their residence and job locations in such a way that they would result in an average trip distance which will satisfy their daily trip rate, within the daily total trip distance as constrained by the travel money and time budgets.

(2) Hence, if travel speeds increase, and the urban area is allowed to expand freely, the difference between the two spatial distributions will increase, namely population will tend to disperse away from the concentration of jobs in and near the center. Conversely, if travel speeds decrease, the travel distance will have to decrease as well by:

(i) relocation of residences back towards the center;
(ii) movement of jobs, to follow the dispersed population; or
(iii) a combination of both.

Actual observations over time indicate that people prefer the relocation of their jobs over the relocation of their residences under congested travel conditions, thus putting into action a process of job dispersal. This mechanism may explain the observation that congested travel conditions tend to change mono-nucleated cities into multi-nucleated ones, where the two spatial distributions, of population and jobs, become more evenly distributed over the area.

(3) It may also be inferred that cities with low levels of motorization and, hence, low total average weighted speeds, will tend to be relatively more compact and with more even distributions of population and jobs than cities with higher motorization levels and travel speeds. Indeed, this seems to be the case when comparing cities in developing vs. developed countries, such as Bangkok vs. London in Figure 5.1.

(4) Conversely, in cities of relatively small size, where travel distances are inherently short, the difference between the two distributions may be more pronounced, such as in Bristol, in Figure 5.1.

The above indications raise two important issues:

(a) In most urban models the location of workplaces are decided first and residences are then allocated accordingly, based on the assumed transportation system. However, if a causal model is considered, it seems that the systematic steps to apply (although with feedback loops) should be: (a) assume the transportation system first; (b) derive the expected changes in population and employment distributions in response to the transportation system; (c) consider the time lag (reaction time) while workplaces adjust themselves to the new condition.
Figure 5.1: Differential Accumulation of Population vs. Jobs, in Percent, by Distance from the City's Center.
The D.A. procedure combines steps (b) and (c) together, although with no time-lag as yet. The next phase of development, therefore, will have to consider this additional factor.

(b) Even if employment centers are planned first, as is the case in some cities of developing countries, special care should be exercised while allocating residences around them, since the transportation system and the future increase in motorization may disrupt the original planning assumptions and start further dispersion around the new centers, thus expanding the city size more rapidly than envisaged.

In conclusion, the D.A. process seems to be a practical tool for defining the interaction between the spatial distributions of population and jobs and the travel conditions in the urban area, as well as for predicting changes in urban structure in response to changes in the transportation network. Several examples of its actual application are described in the following section.

5.5 Application of the D.A. Function

The first possible application of the D.A. function is to rank cities by their urban structure, as detailed in Appendix 7 and shown graphically in Figure 5.2

(See Page 52)

As can be seen, when the cities are described by their \( A \) and \( B \) parameters, they tend to follow definite trends, which seem to apply not only between cities but also over time. For instance, Washington, I.C. is represented by point 8 for 1955 and point 9 for 1968, and it becomes apparent that the change of urban structure over time shifted the place of Washington, D.C. in a predictable way along the two relationships in Figure 5.2.

The above results suggest, therefore, that each city may be defined by its urban structure, compared on an inter-city ranking scale for its relative characteristics, and for its average trip distance, by knowing only the distributions of its population and jobs in the area.

The second possible application of the D.A. process is for predicting the effects of alternative transportation systems on urban structure. The procedure can be described by the following steps:

(1) Derive the expected change in speed by the alternative transportation plan, by the Alpha-relationship (Ref. 7, chapter 2);

(2) Derive the expected change in the average trip distance, within the equilibrium condition of travel and the cross-elasticities of trip distance vs. trip rate;
1. Athens, 1962
2. Baltimore, 1962
3. Bangkok, 1972
4. Bristol, 1966
5. Kuala Lumpur, 1973
7. Tel Aviv, 1965
10. Twin Cities, 1958
11. Twin Cities, 1970

Figure 5.2: Ranking Cities by the Parameters of the Gamma Function.
(3) Apply the relationships in Figure 5.2 for deriving the expected change in the size of the city and the D.A. distribution within it.

Such an example is presented in Figure 5.3, (Appendix 7) where the spatial distributions of population and jobs in Washington, D.C., in 1955 and 1968, are in relative equilibrium with the transportation system and travel demand.

![Graph showing Differential Accumulation of Population vs. Jobs by Distance from the Center, Washington, D.C., 1955-1968](image)

Figure 5.3: Differential Accumulation of Population vs. Jobs by Distance from the Center, Washington, D.C., 1955-1968

An additional example of the D.A. application is the evaluation of alternative land use plans in Bangkok, 1990, as presented in Figure 5.4. Three plans have been considered: Plan G - the official Greater Bangkok Plan, which calls for concentrating commercial land uses in and near the center, and the dispersion of population outwards; Plan U - natural growth of the city, following past trends; and Plan P - a Polycentric development, based on a balanced dispersion of population and jobs.

Figure 5.4 shows how the three alternative plans are expected to affect the need for travel in Bangkok, compared with the 1972 base conditions. The D.A. curves are expressed in absolute values of workers vs. jobs, indicating that:

(1) The G plan will put the heaviest burden of travel on the population, requiring an extensive development of the transportation network (of both roads and transit); while Plan P is the best, traffic-wise;
(2) Plan U and P also indicate the start of regional development, where many jobs are to be located outside the urban area, thus attracting travel outwards, rather than having all work trips moving towards the center. This trend is indicated by the negative values of the D.A. curves near the fringe of the city.

(3) The D.A. process also produces the average trip distance in each alternative, as well as the expected travel intensity (namely, the number of trips per unit length of rings at decreasing radii), thus allowing a first approximation estimation of the required transportation system even before applying the complete traffic model.

A more complex application of the D.A. process is described in the next chapter, where the distributions of population and jobs interact with all other factors by feedback loops.

5.6 In Conclusion

The D.A. is an empirical process. Although its mechanism can be expressed and verified by mathematical analysis, there is as yet no explanation why people tend to behave in the observed manner (experts in human systems could, possibly, contribute to a better understanding of the mechanism). Nonetheless, there is enough evidence to suggest that the D.A. process can be applied as a useful tool for both defining urban structure in quantified terms and describing the expected changes in urban structure in response to changes in the transportation system, within the travel equilibrium conditions in the city.

While mentioning equilibrium conditions, it should be remembered that the travel and land-use system strives to reach an equilibrium under continuously varying factors, such as income, number of vehicles and resulting speeds, and increase in population. Hence, the more dynamic the changes in the factors, the farther are travel conditions from equilibrium. This problem is especially acute in cities of developing countries, which may double their population in a decade by the migration of poor newcomers who cannot afford housing or transportation. It is of importance to note, therefore, that the D.A. function, as such, cannot reflect directly the lack of travel equilibrium because of two principal reasons:

(i) No data were readily available on the number and distribution of workers and, therefore, households or population had to be used instead, assuming that they represent the proportions of workers and that the number of workers equals the number of jobs. While the first assumption may lead to only a slight distortion in the results, the second assumption may mask substantial unemployment, which should be considered as representing latent, unsatisfied, travel demand;

(ii) The application of the travel equilibrium equation has already indicated that many tripmakers, even those who are employed, are far from being in equilibrium, especially transit passengers. Hence, while the D.A. function describes the end result of all factors, it does not differentiate between them.
FIGURE 5.4: Differential Accumulation of Population vs. Jobs by Distance from the City's Center, Bangkok - 1972 and 1990 Alternatives

The above two aspects of the D.A. process, as developed up to now, indicate that: first, the application of the D.A. process will have to be based on workers, and not on their surrogates and, secondly, that the D.A. process has to be integrated with travel demand by feed-back loops, to allow it to approach equilibrium conditions by iteration. The last subject is further elaborated in the next chapter, where the major components of the UMOT model and their interactions are described and discussed.
VI. Structure of the UMOT Model

6.1 Introduction

The structure of the UMOT model is shown by a flow chart in Figure 6.1. There are several possible ways for describing it, such as longitudinally or horizontally by subject, by activities, or by inputs and outputs. Most of these possibilities will be covered briefly, while more attention will be given to the feed-back loops within it, which bring all the interacting factors into equilibrium.

6.2 Inputs and Outputs

The inputs and outputs follow the three principal parts of the model, as detailed below:

(1) Population: The inputs include the standard population characteristics, such as the spatial distribution of households, size, income, workers per household and availability of private vehicles, as well as additional data on the number of trip-makers per household and the daily travel expenditures in money and time. The final outputs are the daily person miles of travel by mode, number of trips and trip rates, trip distances, trip purposes, and trip time and distance frequency distributions. Another output, though by iteration, is the motorization level, which is an important check on the assumed motorization for a future alternative.

(2) Transportation System: The input of the transportation system supply is described by the road network length (by road class) and the transit line length by mode and capacity. An additional input is the number of private vehicles, coming from part 1 above, and the number of commercial vehicles and transit vehicles, all of which use the road network. The outputs are the daily average travel speed of vehicles, private and public, as well as their daily vehicle-miles of travel (VMT).

(3) Urban Structure: The description of the urban structure includes the spatial distribution of households, workers and jobs, preferably by income and job level (such as white and blue collar), as well as the physical and regulatory limitations on the future expansion of the urban area. Additional data, such as the rent distribution in the city, are desirable but not essential. The outputs are the differential accumulation of workers vs jobs, and estimates of the average trip distance and relative amounts of travel that will be generated by the alternative urban plans. One additional output, though
not a specific one at this stage of the UMOT development, is the rent distribution in the city, and this may be added at a later stage.

The interactions between the above three parts by feed-back loops will be discussed later.

6.3 Inputs Subject to Policy Changes

Although each one of the inputs can be changed at will in the future alternative plans, there is one major component in each of the above three parts of the UMOT model that is of particular interest to the policy maker, as follows:

(1) Unit Cost of Travel: The unit cost of travel may be changed on a general or a local basis, such as increasing the price of fuel in the whole country or increasing parking charging in the city center only.

If the change is general, its effect will be widespread, changing the total amount of travel, while if the change is local, its direct effect will also be localized, although the repercussions may spread to other parts of the city. For instance, introducing road charging in the city center may not necessarily change the total amount of travel in the city, but it may well change the patterns of travel in the city, with possible long-range changes in the urban structure. Thus, a change in the unit cost of travel has to be specified, since its degree of impact will vary with different levels of income, and other such household related factors.

The main point to note, though, is that the unit cost of travel is a major variable in the UMOT model, subject and sensitive to policy changes.

(2) System Supply: Although the process of evaluating alternative transportation systems is well known and established, there are a few additions in the UMOT model over standard procedures that are of interest. First and foremost is the ability to test rapidly and easily the global effects of each alternative even before applying a sophisticated traffic model; Secondly, since travel demand is sensitive to system supply in the UMOT model, each system alternative will result in its own level of mobility and modal splits; and thirdly, system supply will also indicate the possible changes in the urban structure. Thus, the UMOT model is a dynamic one, where a change in each one of the inputs will affect and change all the outputs.

(3) Urban Structure: Alternative plans of urban structure can be evaluated either by separate parts of the UMOT model, or by the complete package, with iterations. For
instance, the differential accumulation can be applied first, in order to define major differences between the alternative plans, and eliminate those that result in extremely heavy travel requirements. The complete model may then be applied to the selected alternatives. Furthermore, it is also possible to impose constraints on the urban structure, such as limitations on city expansion, and then test their effects on travel and on adjustments in the land uses in the city.

As can be seen from the above, the UMOT model lends itself to flexible application, where the effects of a change in each input can be assessed either separately or in combination with other inputs. Hence, the planner can have a better insight into, and control of, each planning variable.

6.4 Feedback Iterations

The equation of travel equilibrium (Eq. 3.2) has already indicated that tripmakers strive to maximize their travel distance within their money and time constraints, and that the system supply will determine the proportion of households, by class, that can reach equilibrium between their travel demand and actual travel. This process is carried out by feedback loops, where the outputs of major steps are fed back as new inputs. For instance, if the city is allowed to expand and the system supply follows increases in income, then the equilibrium process will automatically increase the level of motorization toward maximum satisfaction of travel demand. If, however, city expansion and system supply are both constrained, then the increase in the cost of car travel will inhibit the increase in motorization. Hence, motorization levels become sensitive to both system supply and cost of travel.

Basically, there are two feedback processes: the first is for short-range iterations, where given, or assumed, urban structure and system supply will affect the amount and patterns of travel, while the second one is for long-range iterations, where a possible lack of equilibrium between travel demand and system supply will affect urban structure.

The various optional feedback loops are shown in Figure 6.1.

It should be noted at this stage that Figure 6.1 presents the UMOT structure for private transport, since households (and their tripmakers) owning a car have the best opportunity for reaching travel equilibrium. Hence, private travel is analyzed first at this stage of the UMOT development, while transit travel is analyzed separately. Nonetheless, transit travel will be fully integrated within the feedback loops in the next stage of the UMOT development.

6.5 The Perceived Value of Trip Rate

The daily trip rate results from the ratio of the total daily travel distance over the trip distance. It has been shown above that the total daily travel distance results from the two principal travel budgets, of money and of
Figure 6.1: Schematic Flow Chart of the Unified Urban-Transport Model (UMOT) (Private Transport)

A. Population
- Population Characteristics
  - HH Income
    - Motorization
      - Tripmakers
    - Travel Budgets
      - Cost
      - Time
      - Unit Cost
      - Daily Travel Distance
        - Speed
        - Equilibration
        - Trip Rate
        - Trip Purposes
          - Perceived Value of Travel
            - Trip Frequencies

B. Transport System
- System Supply
  - Exp.
  - Art.
  - Loc.
- Private Vehicles
  - C.V.
  - Other
  - The Alpha Relationship
    - VMT
  - Trip Distance

C. Urban Structure
- Urban Structure
  - Physical Limitations on Expansion of Area
- Population Spatial Distribution
- Jobs Spatial Distribution
- Differential Accumulation
- Rent Distribution

Short Range Iterations

Long Range Iterations

== Inputs sensitive to alternative policies
--- Outputs
time, while the average trip distance results from the urban structure. Moreover, tripmakers seem to prefer to increase their trip distance over trip rate when speeds increase. Furthermore, it was also shown in a previous report that the perceived value of trip rate increases at decreasing rates. When combining all these observations, it may be inferred that when speeds increase, tripmakers will prefer to increase their trip distance and, as a long-range result, will also increase the differentiation between the spatial distribution of population and jobs. If, however, speeds decrease, tripmakers will then prefer to decrease their trip rate, rather than decrease their trip distance, and if speeds continue to fall, and city size continues to increase, then jobs will start to follow population, decreasing the differentiation between the two spatial distributions in order to decrease the trip distance or, at least, keeping it unchanged. However, although tripmakers prefer trip distance over trip rate, they become increasingly reluctant to decrease their daily trip rate below a certain minimum value. At this stage, the minimum daily trip rate (averaged for a city) seems to be 2.7 trips per car and 2.0 trips per transit tripmaker.

The above relations are part of the UDOT model, where the perceived value of travel is applied as an additional constraint for the feedback process, as shown in Figure 6.1. Hence, if the road network does not follow population (and car) increase, the effect will be double-pronged: to inhibit the increase in motorization and to decrease the differentiation between the spatial distributions of population and jobs.

6.6 In Conclusion

The UDOT model, as expressed in Figure 6.1, is a simplified presentation of the complex mechanism of activities taking place in a city. It represents a compromise between the temptation to make the model more encompassing and sophisticated, with the risk of increasing its data requirements and the uncertainties in its predictions, and between the urgent need for a simple, though sensitive model, that can not only describe but also explain travel demand as an interacting component with system supply and urban structure.

Conceptually, the UDOT differs from other models in one important aspect: while most models start with a set of comprehensive assumptions which, because of their complex interactions, have to be simplified in order to bring them down to the level of available data and practical calibrations, the UDOT starts with the minimum available data and is then built up by adding empirical relationships within a consistent framework. Hence, the present structure of the UDOT is a framework which can be better elaborated as the need arises. For instance, most models regard peak-period travel

1/ Ref. 7, Figure 3.19.
as an isolated part of the daily travel, in the sense that peak-period travel does not affect the amount of the total daily travel, and vice versa. Furthermore, the possible spread of base-year peak-hour travel into a future peak-period travel is an exogenous assumption, which can easily double or halve the assigned number of trips on the network. In the UMOT model, on the other hand, the total daily travel per tripmaker is constrained by his two principal travel budgets in such a way that if all workers should try to travel to work at the same time, the resulting congestion would take up the major part of their two budgets for just one trip, thus forcing them to disperse their travel over time until congestion decreases to the level where they can satisfy their total daily distance travelled and their daily trip rate. In other words, in the UMOT model, the spread of peak hour into peak period travel is an output of, rather than an input to, the model.

In conclusion, the UMOT model is flexible enough for much further development, as suggested in the next chapter.

---

1/ The total number of trips to work can be assumed to take place in a future alternative during 1, 2 or even 3 hours, thus changing the amount of assigned travel per peak hour.
VII. Conclusions and Recommendations

7.1 The UMOT vs. Other Models

The UMOT model differs from the standard operational traffic and urban/traffic models in many respects, the foremost which are:

(1) The standard models assume that travel demand and system supply are in equilibrium conditions (at least on a daily basis), while the UMOT suggests that it is rather the lack of equilibrium between the two which is one of the major driving forces that changes and shapes urban structure.

(2) The standard models are based on the concept of single trips, which are then aggregated in various ways, such as trips per household or per person, trips by origin and distinction, by mode, by purpose and by time of day. Hence, when the analyses and forecasts are based on single trips, many uncertainties tend to accumulate along the process and the outputs become open-ended, with no sure way of ascertaining the reasonability of the final totals. The UMOT, on the other hand, starts with basic behavioral travel relationships producing controlling totals, while single trips are the final outputs under changing travel and city conditions.

(3) Most of the standard models use economic principles in fragmented ways, such as the effect of income on the generation of households' trips, and the effect of travel cost in the "generalized cost" method for mode choice of single trips, while the cost of single trips and income levels are not linked together in the same process. The UMOT model, on the other hand, applies economic principles in a consistent way from the macro down to the micro levels.

(4) While mentioning the "generalized cost" concept, whether based on money or time, it should be noted that since it is based on the aggregation of times and costs in a linear form, it cannot distinguish between extreme travel cases if their total cost and time are the same. The UMOT resolves this problem by suggesting that the interaction between the two travel budgets is not linear.

(5) Standard models are based on the concept that increases in speed result in saved travel times and, therefore, most of the benefits derived from an improved transportation system are calculated on the basis of the value of such saved times. However, since all available observations suggest that most, if not all, of the saved travel times are traded off for more travel, it can be inferred that the standard models tend to underestimate forecasts of both the amount of travel (induced travel) and the money expenditure allocated to travel. For instance, a recent transportation study in the city of a developing country concluded that a rapid rail
system will be justified on the basis of the travel times it will save, while, at the same time, the estimated total number of person trips with the system is less than without it! The UMOT model, on the other hand, suggests that the benefits from an improved transportation system should be measured by its ability to decrease the travel disequilibrium condition in the city.

The purpose of the above example is to point out a few of the objective difficulties found in the standard urban/transportation models, some of which are discussed intensively in the technical literature. Indeed, one difficulty is already on the verge of being solved, namely the interaction between system supply and urban structure, as found in the new generation of urban models. It is believed, therefore, that the UMOT model may add its share to a better understanding of the travel mechanism, while remaining relatively simple and rapid in application.

7.2 Recommendations

The recommendations may be summarized very briefly by the following points:

1. A plea for deriving the necessary data for establishing the travel money and time budgets in cities of developing countries. Verifying the indication that travel budgets in such cities are much higher by about 50 percent, at least, than in cities of developed countries, is a major need by itself, with far-reaching implications to other fields than transportation, such as to housing projects and the expected households' allocation for rent. Most of the necessary data are available, though well hidden, in data files of transportation surveys, such as the one conducted recently in Singapore.

2. There is also the need to test the UMOT model on more cities than the two examples in this report (Washington, D.C. and Kuala Lumpur), in order to verify its capabilities beyond any doubt. It could therefore, be applied to additional cities in which the Department is interested, such as Istanbul, San Jose, and Abidjan, in order to test its ability to estimate conditions and forecasts of both travel and urban structure over the whole city.

3. Parallel to further development, the present UMOT model can also be put to immediate test in assessing the implications of alternative locations of proposed residential or industrial projects in urban areas and their required transportation systems (such as rapid transit), based on the present inputs already used by sophisticated and complex models. Since the UMOT is simple and rapid in application, the additional money, time and
effort required to apply it in on-going specific urban/transportation projects are negligible when compared with the initial costs of such studies.

(4) The abundance of indications and suggestions that could be derived from the first stage of the UMOT development, especially with respect to the interaction and responsiveness between travel demand, system supply and urban structure, point out to its potentialities for further development, such as the integration of rent within the fundamental equation of travel equilibrium. It also lends itself to regional application as well, thus widening the scope of its usefulness to additional departments in the Bank.
REFERENCES

1. 'Summary of National Transportation Statistics', U.S. Department of Transportation, June 1975, Table 11.


3. Personal correspondence with Dr. M.J.H. Mogridge, Greater London Council, UK.

4. 'Changes of Travel Characteristics Over Time in Two U.S. Cities', by Y.Zahavi in association with Creighton, Hamburg and Assoc., in preparation for the U.S. Department of Transportation.


8. 'Cost of Operating an Automobile (Suburban based operation)', U.S. DOT, FHWA, April 1972.


APPENDICES

1. Unit Cost of Car Travel in the U.S.
2. Cost of Car Travel in Washington, D.C. 1968
4. Person Miles of Travel by District, Washington, D.C. 1968
5. Estimation of Travel in Kuala Lumpur, 1972
6. Two Reports by Mr. James M. McLynn:
   6.1. Equilibrium and Budget Constraints: The Equilibrium Equation;
7. The Differential Accumulation Technique
8. Travel Demand vs. System Supply, K.L. 1972 and 1980
Appendix 6

Two Reports by Mr. James M. McLynn:

6.1. Equilibrium and Budget Constraints: The Equilibrium Equation;
Memo – May 31, 1976
To: Yacov Zahavi
From: Jim Mc Lynn

Subject: Equilibrium and the Budget Constraints: The Equilibrium Equation

1.1 Purpose

The purpose of this memorandum is to examine the meaning of the equation \( \frac{E(I)}{H(I)} = v_c(v) \). \( E(I) \) and \( H(I) \) are the household budgets of money and time, respectively, that are allowed for travel by an average household having income \( I \). The average speed of travel \( v \) is supplied by the road network and activity structure at a cost of \( c(v) \) measured in dollars per mile. The cost function is not a constant, and explicitly depends on the speed of travel. In the following, some simple optimization problems will be described, and their solutions will be related to the given equation.

1.2 Definitions and Notation.

The income \( I \) will be understood to be the daily income, in dollars, of the household. \( E(I) \) defines the portion of the income that the household is willing to expend on transportation. No specific form is assumed for \( E(I) \) other than that it is positive and non-decreasing. The money budget for travel, \( E(I) \), is not assumed to apply to any specific household, but is rather an average for all households in the same income class.
The time budget, $H(I)$, is the total hours of person time that an average household of income $I$ is willing to allocate to travel. The household may have more than one trip maker, and the trip rates may vary as well as the time duration of the trips. Again, Zahavi's work supports the assumption that there is a time budget for households, and he provides estimating relationships derived from the data. No particular form for $H(I)$ is assumed here, other than that it is positive and non-decreasing.

If a household produces $x$ miles of travel during the day, and consumes $t$ hours in doing so, then the average speed of travel, $v$, is given by $x/t$. No trip may have been made at this speed; indeed, no trip is likely to be made at constant speed. The average speed $v$, while applying to no particular trip, is still useful for describing aggregate trip making.

The cost of travel is assumed to be related to the speed of travel; that is, faster modes cost more per mile than slower ones. Again, this is an average cost, and there may be specific routes in a particular city where a faster mode is cheaper than a slower one (which should drive the slower one out of the market in time). The cost of travel per mile will be written as $c(v)$ to indicate its dependence on speed. This is somewhat of a simplification if standing or fixed costs are considered. To illustrate the point, consider automobile costs per mile. Let $A$ be the average daily fixed cost of owning an automobile, and assume that the operating cost per hour is a function of the speed at which it is operated, say $r(v)$. The function $r(v)$ is positive and increasing, since the energy costs alone go up about as the square of the speed.
Allocating the fixed cost over the time of operation, we get
\[ \frac{A}{t} + r(v) \] as the cost per hour of operating the vehicle at speed \( v \). Correspondingly, the cost per mile of operating the vehicle for \( x \) miles at speed \( v \) is given by \[ \frac{A}{x} + \frac{r(v)}{v} \]. Thus, the cost per mile is not simply a function of the speed, but also depends on either the time or distance of travel, provided standing costs are included in the analysis. The data, however, indicate that the hours of operation of an auto are fairly constant within a city, so that the term \( A/t \) is a constant, and the operating costs can be fairly represented as a function only of \( v \). Since the hours of auto use vary between cities, the standing costs will be retained in our analysis.

2.1. An Optimization Problem.

Household travel presumably is conducted for the purpose of producing benefits for the household. While these benefits may be hard to measure, the amount of travel, at least in theory, is not. If \( x \) is the amount of daily travel produced by a household, we can assume that the benefits can be represented by some function \( F(x) \). \( F(x) \) can be assumed positive and increasing with distance. That is, more travel produces more benefits, although perhaps at a decreasing rate. A household then can maximize its benefits by maximizing its travel. But travel is not free; it consumes both money and time. The household then can be thought of as facing a constrained optimization problem; it wishes to maximize its benefits, subject to time and cost constraints. The latter can be expressed in terms of the money and time budgets,
E(I) and H(I). Thus, we can think of a household as trying to choose two things; an amount of travel x, and a time of travel t, such that it maximizes benefits, subject to money and time constraints. Forming the Lagrangian L, we have

\[ L = F(x) + \lambda [E(I) - A - \ln(t)] + \mu [H(I) - t], \]

where the first term in brackets is the money constraint, and the second bracketed term is the time constraint. The equilibrium conditions are given by

\[ \frac{\partial L}{\partial x} = f(x) - \lambda r'(t) = 0 \]
\[ \frac{\partial L}{\partial t} = \lambda \left[ \frac{t}{c} r'(t) - r(t) \right] - \mu = 0 \]
\[ \frac{\partial L}{\partial \lambda} = E(I) - A - \ln(t) = 0 \]
\[ \frac{\partial L}{\partial \mu} = H(I) - t = 0, \]

where we have written \( f(x) \) for \( F'(x) \).

The first two equations are sufficient to determine x and t as functions of the Lagrange multipliers \( \lambda \) and \( \mu \). The second two equations then can be used to find \( \lambda \) and \( \mu \) as functions of \( E \) and \( H \). Finally, x and t can then be written as functions of \( E \) and \( H \), all assuming that the given functions are such that the problem has a solution. Assume that the problem does have a solution, and that the equilibrium values of x and t are given by \( x_0 \) and \( t_0 \). Then
\[ E(I) = A + t_0 r\left(\frac{x_0}{t_0}\right) \]

\[ H(I) = t_o, \]

which together give

\[
\frac{E(I)}{H(I)} = \frac{A}{t_0} + r\left(\frac{x_0}{t_0}\right) = v_o \left[ \frac{A}{x_0} + \frac{r(v_o)}{v_o} \right] = v_o c(v_o, x_o).
\]

This last says that if the household is located in the urban area in such wise that the average speed \( v_o \) is obtainable within the cities' traffic constraints, and if both time and money budgets are met, then the equilibrium condition holds.

To examine this in more detail, but for a simplified problem, let us assume that \( F(x) = x^2 \). That is, benefits are proportional to the area that can be reached. Also assume that \( r(v) = \alpha v^2 \) where \( \alpha > 0 \). The last says that operating costs are proportional to energy, a simple, but not unsatisfactory, assumption. Then the problem reduces to

\[ L = x^2 + \lambda \left[ E - A - \frac{ax^2}{t} \right] + \mu [H - t]. \]

The equilibrium equations are given by

\[
\frac{\partial L}{\partial x} = 2x - 2\alpha \lambda \frac{x}{t} = 0
\]

\[
\frac{\partial L}{\partial t} = \alpha \lambda \frac{x^2}{t^2} - \mu = 0
\]

\[
\frac{\partial L}{\partial \lambda} = E - A - \frac{ax^2}{t} = 0
\]

\[
\frac{\partial L}{\partial \mu} = H - t = 0.
\]
The first equation says that $t = \alpha \lambda$, and the second that $x^2 = \alpha \lambda \mu$. Substituting these values in the last two equations, we obtain $\mu = \frac{E - A}{\alpha}$ and $\lambda = H/\alpha$, and, finally, $x_o = \sqrt{\frac{H(\lambda - A)}{\alpha}}$, $t_o = H$, and $v_o = \sqrt{\frac{(E - A)\lambda}{H}}$. Clearly, a necessary condition for the solution to exist is that $E(I) > A$, i.e., the money budget for travel must exceed the fixed cost of owning an automobile; otherwise, no car, and no car travel.

The foregoing example is purely illustrative, and the cost function assumes the ownership of an automobile, and the operating cost is proportional to the square of the speed of operation. In the real world, the speeds available are conditioned by a number of factors, including the road network, congestion, and speed limits. The optimum speed $v_o$ may not be possible within the system, and then the equilibrium point cannot be obtained. A more realistic example can be provided by using the empirical equation developed by Zahavi which gives $c(v)$, the cost per mile of travel, as

$$c(v) = 1.68v^{-0.75}.$$ 

Using this cost function, the Lagrangian becomes

$$L = x^2 + \lambda [E - \alpha x v^{-\frac{2}{3}}] + \mu [H - t],$$ 

where we have written $\alpha$ in place of the constant 1.68. The equilibrium equation becomes

$$\frac{\partial L}{\partial x} = 2x - \frac{\alpha \lambda (t)}{v} \frac{1}{x} = 0$$

$$\frac{\partial L}{\partial t} = -\frac{3}{4} \alpha \lambda (\frac{x}{t})^{\frac{1}{4}} - \mu = 0$$
\[ \frac{\partial L}{\partial \lambda} = E - \alpha x^\frac{1}{4} t^\frac{3}{4} = 0 \]
\[ \frac{\partial L}{\partial \mu} = H - t = 0. \]

At equilibrium, we have
\[ x_o = \frac{E^4}{(\alpha^4 H^3)} \]
and
\[ t_o = H, \]
so that
\[ \frac{E(I)}{H(I)} = \alpha \left( \frac{x_o}{t_o} \right)^{\frac{1}{4}} = \alpha V_o^{\frac{1}{4}}. \]

This last says that the equilibrium speed is given by
\[ V_o = \left[ \frac{E(I)}{\alpha H(I)} \right]^{\frac{1}{4}}, \]
provided that this speed is available from the transportation system. As an example, assume that for some income class the money budget is $8.00 per day, and that there are 2.2 trip makers in the average household of that income. Using \( \alpha = 1.68 \) and one hour for the travel time budget per trip maker, we get an equilibrium speed of about twenty-two miles per hour. For a money budget of $6, 1.8 trip makers, and a one hour travel time budget, we get a speed of about fifteen miles per hour.

From the statement of the general problem and the two examples, it should be clear that in those cases where a proper maximum exists, the two budget constraints are sufficient to determine
and a generalized time of

\[ T = t + \frac{c}{\lambda}, \]

where in each case \( \lambda \) is the value of time in cents per hour. Travel demand models have been developed and calibrated, using one or the other of these measures as the only form in which the time and cost variables enter the model. The drawback to this approach is that the generalized measures cannot, even in extreme cases, differentiate between markedly different alternatives.

A second problem associated with the generalized times and costs revolves around which one is "better". For a single trip, one is simply a multiple of the other. From

\[ T = \frac{c}{\lambda} + t = \frac{c + \lambda t}{\lambda} = \frac{c}{\lambda}, \]

we see that the generalized cost is simply the value of time \( \lambda \), multiplied by the generalized time. The \( \lambda \)'s, however, depend on income and, consequently, weighted sums and weighted averages (weighted by the populations in the income groups) will not preserve the simple relationships. In comparing two ways of making the same trip (with different times and costs), the two generalized measures can produce opposing evaluations of the alternatives. This situation has led to lengthy discussions of which is the better measure and under what conditions. (The notion that neither is acceptable seems to be rejected out of hand.) Much of the discussion centers on whether it is preferable to assume constant marginal utility of time or constant marginal utility of money. The two budget approach does not require the
assumption that any marginal utility is a constant with respect to income. If the benefit function \( F(x) \) of the preceding paragraph is regarded explicitly as a function of income, say \( F(x, I) \), and further assumed to be a utility scale, its equilibrium point, subject to the twin budget restrictions, can be found, provided only that \( \frac{\partial F}{\partial x} > 0 \) and the cost function is reasonably well-behaved. In this sense, at least, the two budget approach can be regarded as richer and more versatile than the generalized value of time or cost approach.
Memo - May 12, 1976

To: Yacov Zahavi
From: Jim McLynn

Subject: The Differential Accumulation Process

1.1 Purpose

In this memo we shall attempt a formalization of the differential accumulation process in order to provide some insight into why it works. In addition, we will provide some slight connection to other models of some interest, again in the hopes of illuminating the process. Finally, we will suggest an alternative method for fitting the gamma distribution.

1.2 Description of the Differential Accumulation Process.

Zahavi begins by describing the location of workers in a city by the distribution of their distances from the city center (which can presumably be located with reasonable accuracy). If worker locations are not available, he uses a surrogate, such as households or population locations. Similarly, he describes the job locations by the distribution of their distances from the center of the city. The differential accumulation process is now defined in the following way. Let $0 < x_1 < x_2 < x_3 < \ldots < x_n$ be the mid-point distances of rings about the center of the city. Let $p(x_i)$ and $e(x_i)$ be the fractions of workers and jobs, respectively, in the ring associated with $x_i$. Define $g(x_i)$ by

$$g(x_i) = p(x_i) - e(x_i)$$
\[ g(x_{n-1}) = p(x_{n-1}) + p(x_n) - e(x_{n-1}) - e(x_n) \]
\[ \vdots \]
\[ g(x_k) = \sum_{j=k}^{n} [p(x_j) - e(x_j)] \]
\[ \vdots \]
\[ g(x_1) = 0. \]

The function \( g(x_1) \) is called the differential accumulation function. From it one can calculate

\[
\frac{\sum_{i=1}^{n} x_1 g(x_1)}{\sum_{i} g(x_i)} \tag{1}
\]

and this quantity turns out to be close to the mean of the journey to work distances in each of the seven disparate cities for which data is given in Zahavi's memorandum*. This remarkable result perhaps overshadows the intriguing fact that \( g(x_1) \) is always non-negative for the cities considered.

1.3 The Gamma Distribution.

Define \( \tilde{g}(x_1) \) by

* Memorandum to George Beier, dated 7 November, 1975.
\( \hat{g}(x_1) = \frac{1}{\sum \hat{g}(x_1)} \)

where \( \hat{g} \) can be thought of as the normalized differential accumulation function. Graphs of \( g(x_1) \) and \( \hat{g}(x_1) \) are plotted in the referenced memo for each of the cities. The memo then suggests that \( \hat{g}(x_1) \) can be well approximated by a gamma distribution defined by

\[
\gamma(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{(\alpha-1)!}
\]

A process (based on maximum likelihood estimation) is described for determining \( \alpha \) and \( \beta \), and interpretations are given. In particular, the mean of \( \gamma(x; \alpha, \beta) \) is \( \alpha/\beta \), and this must be close to the mean of \( \hat{g}(x_1) \), which, in turn, is an estimate of \( \hat{\alpha_w} \), the mean journey to work distance. Somewhat later we will suggest a different technique for choosing \( \alpha \) and \( \beta \) that is a) simpler, and b) more useful for the particular case at hand.

2.1 Formal Description

In this section, we will generalize the differential accumulation scheme somewhat, and then show that it can be described in familiar terms. In particular, we will relate it to the characteristics of the worker and job distribution in a way that will shed some light on the process.

Let \( p(x) \) and \( e(x) \) be the continuous density distributions of workers and jobs as a function of distance from the center of the city. Distance can be measured in any metric whatsoever, and we
make, at this point, no assumptions about the distributions \( p(x) \) and \( e(x) \) other than \( \int_{0}^{L} p(x)dx = \int_{0}^{L} e(x)dx = 1. \)

Define the function \( g(x) \) by

\[
g(x) = \int_{x}^{L} p(t)dt - \int_{x}^{L} e(t)dt
\]

in analogy with the earlier definition of \( g(x_1) \). The first integral gives the number of workers that are located at a distance of at least \( x \) length units from the center. The second integral gives the number of jobs at distance \( x \) or more from the center. The difference, as before, is called the differential accumulation function.

The first thing to observe about (2) is that there is nothing unique about starting with the outer ring. We could just as easily have gone in the other direction since

\[
g(x) = \int_{x}^{L} p(t)dt - \int_{x}^{L} e(t)dt = \int_{0}^{x} e(t)dt - \int_{0}^{x} p(t)dt. \tag{3}
\]

For convenience we will also write \( g(x) \) in the equivalent form

\[
g(x) = 1 - P(x) - (1-E(x)) \tag{4}
\]

where \( P(x) = \int_{0}^{x} p(t)dt \) and \( E(x) = \int_{0}^{x} e(t)dt \). From this last we obtain

\[
\int_{0}^{L} g(x)dx = \int_{0}^{L} [1-P(x)]dx - \int_{0}^{L} [1-E(x)]dx = \mu_P - \mu_E \tag{5}
\]
where $\mu_p$ and $\mu_E$ are the mean distances of workers and jobs from the center.

Equation (5) says that the area under the differential accumulation curve is the difference of mean distances of workers and jobs from the center.

The next term we need to evaluate in terms of the assumed (or observed) distributions $p(x)$ and $e(x)$ is $\int_0^L xg(x)dx$. From (4) we have

$$\int_0^L xg(x)dx = \int_0^L x[1-P(x)]dx - \int_0^L x[1-E(x)]dx$$

$$= \frac{1}{2}\int_0^L x^2p(x)dx - \frac{1}{2}\int_0^L x^2e(x)dx$$

$$\int_0^L xg(x)dx = \frac{1}{2}[m_2(P) - m_2(E)] \tag{6}$$

where we have written $m_2(P)$ and $m_2(E)$ for the second moments about the origin of $p(x)$ and $e(x)$, respectively. From (5) and (6) we have

$$\int_0^L xg(x)dx \int_0^L g(x)dx = \frac{m_2(P) - m_2(E)}{2(\mu_p - \mu_E)}. \tag{7}$$

Analogously to what was done in paragraph 1.3, we define $\hat{g}(x)$, the normalized differential accumulation function, by

$$\hat{g}(x) = g(x)/\int_0^L g(t)dt \tag{8}$$

and from (7) we have that the mean of $\hat{g}(x)$ is given by
\[ \int_0^L x \hat{g}(x) dx = \frac{m_2(P) - m_2(E)}{2(\mu_P - \mu_E)}. \]  

(9)

Thus, if Zahavi's observation that the mean of the normalized differential accumulation function is a good estimate of the average journey-to-work distance \( \bar{d}_W \) is correct, then

\[ \bar{d}_W = \frac{m_2(P) - m_2(E)}{2(\mu_P - \mu_E)}. \]  

(10)

The results (5), (6), (7), and (9) are purely mathematical results, depending only on the definitions (2) and (8). Their effect is simply to convert statements about the moments of \( g(x) \) into terms of the moments of \( p(x) \) and \( e(x) \). This conversion, while trivial, is not an idle one. From (9) we see that if \( \mu_P = \mu_E \) and \( m_2(P) \neq m_2(E) \), then the mean of \( \hat{g}(x) \) is infinite and (10) cannot be true. In general, we see that any underlying model of the travel process is going to have trouble when \( \mu_P - \mu_E \) is small and \( m_2(P) - m_2(E) \) is not. Thus, the model must be carefully circumscribed with respect to its range of validity. Perhaps a more important advantage of the conversion is that equation (10) is more suggestive to our intuition than is its equivalent

\[ \int_0^L x \hat{g}(i) dx = \bar{d}_W \]

in that we are more familiar with means and second moments than with differential accumulation functions.

The right-hand side of (9) or (10) can be rewritten in terms
of the variances by recalling that \( \sigma^2 = m_2 - \mu^2 \), i.e.

\[
\frac{m_2(P) - m_2(E)}{2(\mu_P - \mu_E)} = \frac{\sigma_P^2 + \mu_P^2 - \sigma_E^2 - \mu_E^2}{2(\mu_P - \mu_E)}
= \frac{\mu_P + \mu_E}{2} + \frac{\sigma_P^2 - \sigma_E^2}{2(\mu_P - \mu_E)}. \tag{11}
\]

The first term on the right-hand side is simply the average of the distances of jobs and workers combined from the city center.

If we write

\[ \mu = \frac{1}{2}(\mu_P + \mu_E) \]

and

\[ \sigma = \frac{1}{2}(\sigma_P + \sigma_E), \]

we have

\[
\frac{m_2(P) - m_2(E)}{2(\mu_P - \mu_E)} = \mu + \left(\frac{\sigma_P - \sigma_E}{\mu_P - \mu_E}\right)\sigma. \tag{12}
\]

It is instructive to calculate the two terms of (12) for the cities discussed by Zahavi. In the following table we have in each case used the midpoint of the rings in the calculations. The correction factors for the approximate estimations of the integrals have been omitted, as we are only interested in the relative size of the two terms.
<table>
<thead>
<tr>
<th>City</th>
<th>( u = \frac{1}{2}(\mu_p + \mu_E) )</th>
<th>( \frac{(\sigma_p - \sigma_E)}{\mu_p - \mu_E} )</th>
<th>Est. Av. Distance</th>
<th>Obsvd. Av. Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens</td>
<td>4.71</td>
<td>-0.18</td>
<td>4.53</td>
<td>5.57</td>
</tr>
<tr>
<td>K. L.</td>
<td>6.04</td>
<td>-0.66</td>
<td>5.38</td>
<td>5.53</td>
</tr>
<tr>
<td>D. C.</td>
<td>5.81</td>
<td>-0.19</td>
<td>5.62</td>
<td>6.68</td>
</tr>
<tr>
<td>Tel Aviv</td>
<td>4.72</td>
<td>+0.03</td>
<td>4.75</td>
<td>4.10</td>
</tr>
<tr>
<td>Bankok</td>
<td>6.84</td>
<td>+0.93</td>
<td>7.77</td>
<td>7.70</td>
</tr>
<tr>
<td>Baltimore</td>
<td>9.98</td>
<td>-0.83</td>
<td>9.15</td>
<td>9.33</td>
</tr>
</tbody>
</table>

From the data, it appears that the second term on the right-hand side of equation (12) is generally small, compared to the first term. This suggests that we might interpret Zahavi's differential accumulation process as saying:

The average journey-to-work distance can be approximated by \( u = \frac{1}{2}(\mu_p + \mu_E) \), where \( \mu_p \) is the average distance of workers' residences from the city center, and \( \mu_E \) is the average distance of job locations from the city center. The approximation can be improved by adding a correction term given by

\[
\frac{\sigma_p^2 - \sigma_E^2}{2(\mu_p - \mu_E)}
\]

where \( \sigma_p^2 \) and \( \sigma_E^2 \) are the variances of the distribution of workers' residences and jobs from the city center, respectively.

2.2 A Lower Bound for Average Trip Length.

In this section, we will relate the term \( \mu_p - \mu_E \) to the classical assignment problem, and show that it is a lower bound
for the average journey-to-work distance.

Consider an urban area of finite extent with a population of workers and a population of jobs equal in number. Let $P_1, P_2, \ldots, P_N$ be the points where the workers are located (in any coordinate system), and let $Q_1, Q_2, \ldots, Q_N$ be the points where the jobs are located. Let $d_{ij} = \mu(P_i, Q_j)$ be the distance between the location of worker $i$ and job $j$. The function $\mu$ is any metric whatsoever with the usual properties:

(i) $\mu(P,P) = 0$
(ii) $\mu(P,Q) = \mu(Q,P)$
(iii) $\mu(P,Q) + \mu(Q,R) \geq \mu(P,R)$.

Examples of $\mu$ for rectangular coordinates $(x_i, y_i)$ are

$$\mu(P_i, P_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

and

$$\mu(P_i, P_j) = |x_i - x_j| + |y_i - y_j|.$$

This last metric is sometimes called "Manhattan distance" and corresponds to the distance between points in a city with a rectangular grid of streets.

We now ask what is the minimum distance of total travel necessary to get the workers to the jobs. Assuming the jobs and the workers are respectively homogeneous, we can attempt to find the "best" assignment of people to jobs in the sense that total travel is minimized. This will clearly be no more than the minimum obtained if the jobs and people are respectively differentiated,
since any constraints on the assignment will of necessity impose constraints on the value of the minimum.

The assignment problem can be formulated in the following way:

\[
\begin{align*}
\text{find } x_{ij}, \text{ where } x_{ij} = 0 \text{ or } 1, \text{ subject to} \\
\sum_{j=1}^{N} x_{ij} &= 1 \\
\sum_{i=1}^{N} x_{ij} &= 1
\end{align*}
\]

and such that

\[
\phi = \sum_{ij} d_{ij} x_{ij}
\]

is a minimum.

The 2N equations of constraint express the condition that each person is assigned to only one job and that each job is assigned to only one person. \( \phi \) is the total distance of travel to work.

A lower bound for \( \phi \) can be obtained in the following way. Introduce a point \( C \) corresponding to the center of activity of the urban area (actually \( C \) can be any point, but the city center concept provides the most meaningful interpretation). From the triangle inequality (property (iii) of the metric), we have

\[
\mu(P_1, Q_j) + \mu(Q_j, C) \geq \mu(P_1, C)
\]

and
\[ u(Q_j, P_i) + u(P_i, C) \geq u(Q_j, C). \]

The first of these gives
\[ d_{ij} = u(P_i, Q_j) \geq u(P_i, C) - u(Q_j, C) \quad (13) \]

And the second, using property (ii), gives
\[ d_{ij} = u(P_i, Q_j) \geq u(Q_j, C) - u(P_i, C). \quad (14) \]

Using (13) and (14) together, we have
\[ d_{ij} \geq |u(P_i, C) - u(Q_j, C)|. \quad (15) \]

Substituting (13) in \( \phi = \sum_{ij} d_{ij} x_{ij} \), we obtain
\[
\phi \geq \sum_{ij} \left( u(P_i, C) - u(Q_j, C) \right) \\
= \sum_{ij} \Sigma x_{ij} u(P_i, C) - \sum_{ij} \Sigma x_{ij} u(Q_j, C) \\
= \Sigma u(P_i, C) - \Sigma u(Q_j, C),
\]

since \( \Sigma x_{ij} = 1 \) and \( \Sigma x_{ij} = 1 \). This inequality says that the total distance traveled in the optimal assignment of people to jobs is at least as great as the sum of the worker distances from the center minus the sum of the job distances from the center.

In a similar fashion, using (14), we get
\[ \phi \geq \sum_{j} u(Q_j, C) - \sum_{i} u(P_i, C). \]

Combining this with the previous result, we have
\[ \phi \geq \left| \sum_{i} \mu(P_i, C) - \sum_{j} \mu(Q_j, C) \right|. \]

The average distance traveled is \( \phi/N \), where \( N \) is the number of workers and jobs. So we may write

\[ \frac{\phi}{N} \geq \left| \frac{1}{N_1} \sum_i \mu(P_i, C) - \frac{1}{N_j} \sum_j \mu(Q_j, C) \right|, \]

which says:

the average travel distance to work in an optimal assignment of people to jobs is no less than the absolute value of the difference of the average distance of the workers from the city center and the average distance of the jobs from the city center.

In the usual case, the average distance of the workers from the city center, say \( \mu_P \), is greater than the average distance of the jobs, say \( \mu_E \), from the city center, so that we must have

\[ \frac{\phi}{N} \geq \mu_P - \mu_E. \quad (16) \]

This result provides an interpretation of the term \( \mu_P - \mu_E \), which occurred in the expression for the differential accumulation function given in Paragraph 2.1.

3.0 Fitting the Gamma Distribution.

The objective here is to find, in a simple way, a gamma distribution which approximates the normalized differential accumulation function \( \hat{g}(x_1) \), defined by
\[ \hat{g}(x_i) = g(x_i) / \sum_{i=1}^{M} g(x_i). \]

The mean of \( \hat{g}(x_i) \) is the quantity of interest to us (since it approximates the average journey-to-work distance), and it is therefore appropriate to define the gamma distribution in a manner such that its mean will be the mean of \( \hat{g}(x) \). Thus, if the gamma distribution is given by

\[ \gamma(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{(\alpha-1)!}, \]

its mean is given by \( \alpha/\beta \). If we require

\[ \frac{\alpha}{\beta} = \frac{\sum x_i g(x_i)}{\sum g(x_i)}, \quad (17) \]

then \( \alpha/\beta \) will provide the identical estimate of the mean journey-to-work distance as the discrete differential accumulation function. An additional relation is required to determine the specific values of \( \alpha \) and \( \beta \). A reasonable one is to require that the second moments be identical. Since the second moment of the gamma distribution is given by \( \alpha(\alpha+1)/\beta^2 \), we have

\[ \frac{\alpha(\alpha+1)}{\beta^2} = \frac{\sum x_i^2 g(x_i)}{\sum g(x_i)}. \quad (18) \]

The two equations (17) and (18) are sufficient to uniquely determine \( \alpha \) and \( \beta \) as
\[
\alpha = \frac{[\Sigma x_1 g(x_1)]^2}{\Sigma g(x_1) \Sigma x_1^2 g(x_1) - [\Sigma x_1 g(x_1)]^2}
\]
\[
\beta = \frac{\Sigma g(x_1) \Sigma x_1 g(x_1)}{\Sigma g(x_1) \Sigma x_1^2 g(x_1) - [\Sigma x_1 g(x_1)]^2}.
\]

Thus, \( \alpha \) and \( \beta \) are determined by \( \Sigma g(x_1), \Sigma x_1 g(x_1), \) and \( \Sigma x_1^2 g(x_1), \) the zeroth, first, and second moments of the differential accumulation function, all readily calculated from the data.