DYNAMIC EFFECTS OF ENERGY POLICIES ON TRAVEL BEHAVIOR AND URBAN STRUCTURE

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ABSTRACT

The paper discusses the urgent need for travel models that would be capable of predicting accurately the effects of acute energy shortage, and/or high increases in gasoline prices, on travel behavior and urban structure.

The paper presents new approaches to the understanding of travel behavior. Two models are presented. The first is based on the theory of consumer choice under explicit time and money constraints; the second is a dynamic mode-choice model based on the theory of bifurcations. Such a model is capable of capturing the sudden effects upon travel behavior of smooth changes in travel costs at critical thresholds, that could result in long-term bifurcations in the behavior of urban structure.

The paper concludes with recommendations for further research, with special emphasis on the urgent need to know more on the critical thresholds of travel behavior, where the individual choice-maker interacts both with his fellow residents and with urban structure, and to have more reliable information on the range of relaxation times of the various components of urban structure and activities.

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I. INTRODUCTION

1. It is well recognized in many fields of science, including mathematics, physics and biology, that a group of elements is to be regarded as a system on a higher level than its elements; the whole is more than the sum of its parts. For example, a living biological organism is more than just the summation of the individual characteristics of its cells.

The same concept can also be applied to a city, which can be regarded as a system where individuals form sub-groups at various levels, such as by income, ethnic, employement, and location (neighborhood) characteristics. Thus, and similar to a biological organism, while a city is composed of its individual inhabitants, it is more than just the sum of their characteristics. This concept was already applied in mathematical terms to social systems by Rashevsky [1] and others. A step further along these lines is the work by Prigogine [2] and his co-workers at the University of Brussels [3] on the notion of self-organizing systems.

2. There are two general approaches to the subject of interactions between the elements and their group. One prevalent approach is to deal with the group at a macro, aggregate level. This approach requires to treat the individuals' behavior as micro fluctuations, or perturbations, within the group, along the lines of the "Order by Fluctuations" principle, proposed by Prigogine.

The second approach, followed in this paper, is to start at the micro, disaggregate level of behavior, where the behavior of individuals is assumed to be governed by rational economic principles, and then link it with the group's behavior. This line of research is based on utility maximizing behavior. At this point we meet with the problem of interactions and feedback process between the behavior of the individuals

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and their group. Perhaps the best way of explaining this problem is by an example from mass psychology: when a croud is enticed to act in a violent way, the individual decisions to change from peacful to violent behavior are added-up until a critical threshold is crossed, after which the crowd may explode into a violent mob. The same violent reactions of a group of elements are also well known in chemistry and atomic physics, when a "critical mass" is reached, and the group's reaction accelerates explosively.

3. Referring back to the city, it can be regarded as a complex dynamic interactive system. Let us now assume that one factor, say travel costs, increases continuously over time, thus forcing an increasing number of individuals in a city to alter their mode choice. Let us also assume that modal change at the level of individual traveler/household, say from car to bus, is either continuous or abrupt, and the question is whether the sum of such micro-changes in individual behavior results in continuous macro-changes in travel behavior at the city level, or a certain critical threshold might be reached, after which a noticeable sudden change in the overall city travel pattern is to be expected. Furthermore, would all urban structures, in cities of developed and developing countries, respond in a similar way to rapidly increasing travel costs, or would some structures be more resilient than others, able to absorb higher levels of travel costs before reaching critical thresholds ?

Such questions are urgent especially now, when energy costs increase continuously, and since they are regarded as principal parameters in describing urban structure and activity levels in a city, any significant changes in travel costs may generate powerful forces that can affect and even change cities in unexpected ways.

4. The above questions are too complex to be fully addressed in one paper. Therefore, only a limited number of issues are discussed below, with the emphasis put on the methodology of analysis, rather than on concrete solutions. The main theme of the paper is travel behavior under continuous increases in travel costs, and its possible effects on urban structure. The paper starts with a new approach to the analysis of individual travel behavior, with special attention given to mode choice. It is then suggested that theories of bifurcation can assist us in the identification of critical thresholds in travel behavior at the individual, micro level. Several possible effects of such sudden changes in mode choice on urban structure are then offered for consideration, and the paper concludes with recommendations for specific directions for further research.

II. THE UMOT APPROACH

1. Based on extensive empirical investigations, a new approach to travel modeling has recently been put forward by Zahavi, called the Unified Mechanism of Travel, or UMOT for short [4]. Its main characteristics can be summarized as follows:

   (a) The daily mean expenditures on travel in urban areas, per representative traveler and household, in time and money terms, are found to display predictable relationships. Such expenditures have been found to be transferable both between cities and over time in developing countries. They are therefore regarded as "travel budgets". Therefore, these travel budgets may be applied as explicit constraints on travel behavior. The application of explicit constraints in modeling travel behavior is a powerful tool, since the constraints do away with the need for much of the coefficient calibration in conventional models.

   (b) It was found useful to base the UMOT model on a particular theory of human decision making called utility theory. The basic premise in the UMOT model is that travelers attempt to maximize the utility of their spatial and economic opportunities, as represented by their total average weekday travel distance, within the constraints of their travel time and money budgets. In the UMOT approach, distances traveled are intrinsically associated with access to opportunities to consume and/or produce; this is at variance with the conventional approach to travel distance [4, 5, 6].

   (c) The UMOT model deals with travel behavior at both the macro and micro levels. It is based on the observation that the variations about the mean travel budgets are similar for all socioeconomic groups. Therefore, it is necessary to forecast only the mean values of the budgets for each group. These mean values, together with the variations around them, provide the probabilities of individual travelers behaving in predictable ways.

2. The following example describes in more detail some of the concepts behind the mode-choice process of the UMOT model. It should be noted that mode choice in this case is based upon the daily travel distance by mode, and not - as conventionally done - by single trips. In order to simplify this presentation, the example is presented in mean values, for representative households.

III. CRITICAL THRESHOLDS IN MODE CHOICE BEHAVIOR

1. Appendix 1 summarizes the observed daily travel time and money expenditures per average household, by income, as derived from the 1968 comprehensive home-interview survey in Washington, D.C. The total travel time expenditure per household is the door-to-door travel time as reported by the respondents, and it increases with household income because the number of travelers per household increases with income. The travel money expenditures were derived from the reported travel distance, by mode.

   Appendix 1 also details the 1968 unit costs of travel by car and by bus, as well as by an assumed rapid transit system (rail or bus on their own right-of-way) with an extensive coverage, similar to the bus system. Also shown in the table are the daily travel distances per household that can be generated by each of the three modes observing each travel budget separately (i.e., by dividing each budget by the unit cost of each mode). The last part of the table includes the maximum travel distance that can be generated by the given unit costs observing the two constraining budgets simultaneously by using combinations of the available modes. These values are estimated by the UMOT maximization process, which produces both the total travel distance and the mode choice by distance.

   Figure 1 shows the travel distance by each mode within the constraining budgets, for pairs of modes. Although the three diagrams in Figure 1 are simple in concept, they are rich in implications. For instance:

   (a) The car/bus diagram suggests that households with an annual 1968 income above approximately $ 4,000 can utilize both travel budgets in trade-offs to achieve maximum travel benefits (i.e., maximum travel distance in this case). The area where such trade-offs are possible is shaded, and it is called the choice-set. The maximum possible distance within the choice set is shown by the thick curve. There are cases, however, where one budget alone is binding. For instance, representative households below an annual 1968 income of approximately $ 4,000 are constrained in travel choices by the money budget alone and, hence, have a practical choice of the bus mode only. Representative households above an annual 1968 income of approximately $ 15,500 (beyond the range shown in the diagram), on the other hand, are constrained by the time budget alone and, hence, are expected to prefer the speedier mode, namely car only. Such cases analytically operationalize the planning concepts of mode choice and "captive" riders on particular modes.

   (b) The pair car/rapid transit diagram suggests that the choice-set in this case is shifted to the range of higher incomes. Put another

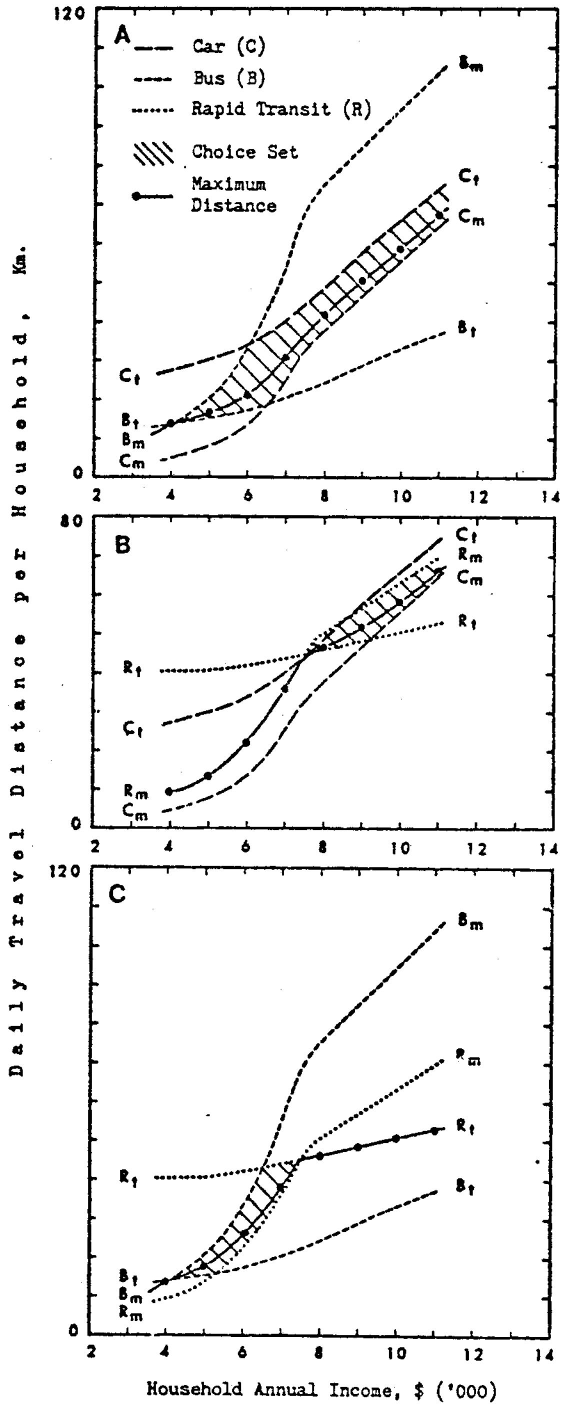

Figure 1 : Mode Choice-Set and Maximum Travel Distance per Average Household Under the Travel Time (t) and Money (m) Budgets, by Mode Pairs, vs. Household Annual Income, Based on Travel Budgets and Unit Costs in Washington, D.C. 1968. (A. Car and Bus; B. Car and Rapid Transit; C. Bus and Rapid Transit).

way, the availability of a rapid transit system lessens the demand for car travel in a city. That is, the relatively high speed of rapid transit makes it an attractive substitute for car for low and medium income households. Indeed, if the speed of the rapid transit system is assumed to be lower than in Appendix 1, it lowers the R_t curve in the diagram, and expands the choice-set (including the demand for car travel) to cover households with lower incomes.

(c) The bus/rapid transit diagram suggests that the choice-set is restricted to relatively low income levels, and that households with income levels above approximately \$ 7,500 are already constrained by their time budget alone.

Figure 2 shows the choice-set for all three modes combined, and it appears that the choice-set expands only slightly when compared with the car/bus case. However, when referring to Appendix 1, it becomes apparent that the addition of rapid transit to the car/bus pair reduces significantly the demand for car travel. For instance, households with an annual 1968 income of \$ 7,000 would require 19.7 car passenger-km in order to maximize their daily travel distance by the car/bus pair, but only 11.3 car passenger-km would be required if a rapid transit mode is added. Thus, the demand for cars is expected to be lower than in the former case.

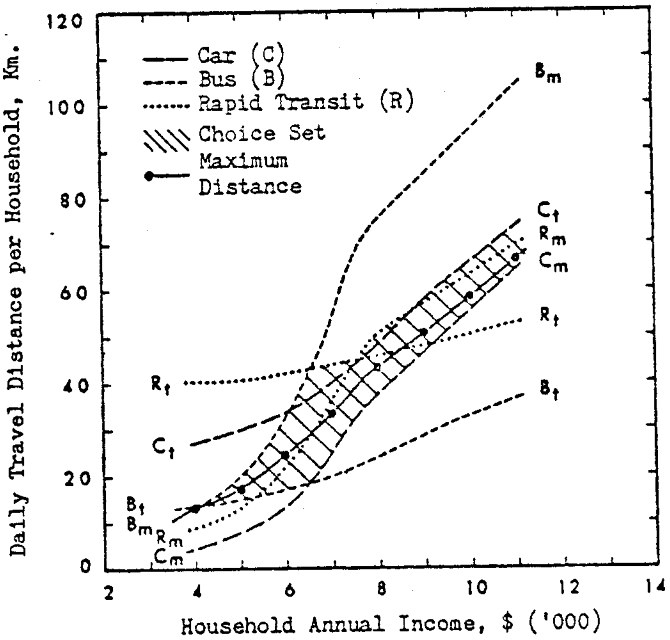

Figure 2 : Mode Choice-Set and Maximum Travel Distance per Average Household Under the Travel Time (t) and Money (m) Budgets, Three Modes, vs. Household Annual Income, Based on Travel Budgets and Unit Costs in Washington, D.C. 1968.

Furthermore, the addition of the rapid transit mode would also enable these households to increase their daily travel distance, from 30.8 to 33.8 passenger-km within their travel budgets. It is also apparent that the rapid transit mode will also attract some travel from the bus mode. In short, the addition of a rapid transit system, with the operational characteristics detailed in Appendix 1, is expected to affect the $ 7,000 household in the following way: (i) a reduction by 43 percent in the demand for car travel, (ii) a reduction by 21 percent in the demand for bus travel, and (iii) an increase by 9.8 percent in the total daily travel distance.

It should also be noted that the above effects are differential, depending on the household's income level (and more generally, on the household's socioeconomic characteristics and location). For example, the above mentioned effects for a household with an annual 1968 income of $ 11,000 are expected to be: (i) a reduction by only 11.7 percent for car travel, (ii) a reduction by 50.5 percent for bus travel, and (iii) an increase by only 1 percent in the total daily travel distance. Namely, the effects of a rapid transit system appear to be more pronounced for low and medium income levels than for high income levels. Such expected results appear to be consistent with available evidence in many cities.

Figures 3 and 4 show the modal split, by distance, for the pairs of modes and all three modes combined, respectively, based on Appendix 1. Of special interest is diagram 3a, where the <u>observed</u> 1968 travel distance by car and bus in Washington, D.C., shown as dots, are superimposed on the <u>estimated</u> travel choices, as continuous curves. The fit between the estimated and the observed values is encouraging, especially in light of the fact that the estimated values were <u>not</u> calibrated to the observed values of trip characteristics, but were derived from the observed travel budgets, unit costs of travel, and theoretical relationships suggested by the UMOT process.

2. Figures 3 and 4 display an important phenomenon, namely the <u>bifurcation of mode-choice</u> at critical points. For instance, Figure 4 suggests that households with annual 1968 incomes above $ 7,000 have the choice of three modes, car/bus/rapid transit. However, below $ 7,000 the choice-set degenerates to two modes, bus/rapid transit, and below $ 4,000 the choice-set degenerates to one mode, bus only. It is also evident that changes in all interacting parameters, namely household income, time budget, money budget, and unit costs per mode, in time and money terms, may change these critical points.

   One possible interpretation of such bifurcation points, following them from right to left in the diagrams, is as follows: within a stable area of the choice-set the household is faced by short-term (fast) decisions of mode choice, namely the use of combinations of modes in order to

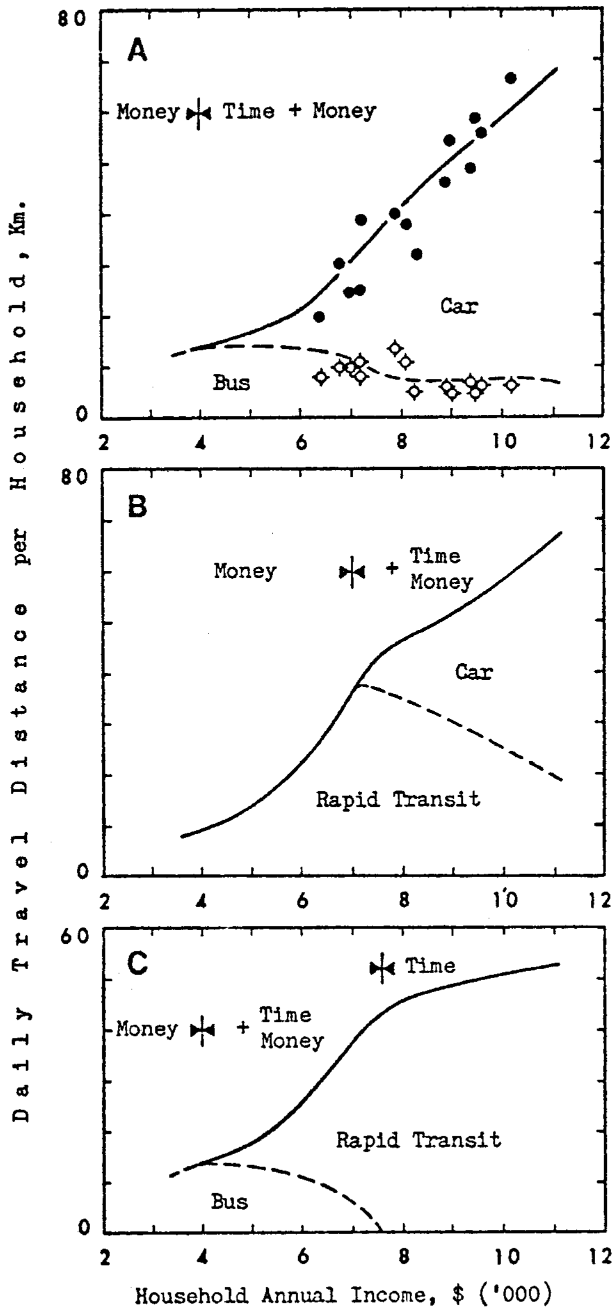

Figure 3 : Estimated Maximum Daily Travel Distance per Average Household, by Mode Pairs, Under the Travel Time and Money Budgets and their Constraining Ranges, vs. Household Annual Income, Based on Travel Budgets and Unit Costs in Washington, D.C. 1968 (A. Car and Bus, including observed values; B. Car and Rapid Transit; C. Bus and Rapid Transit).

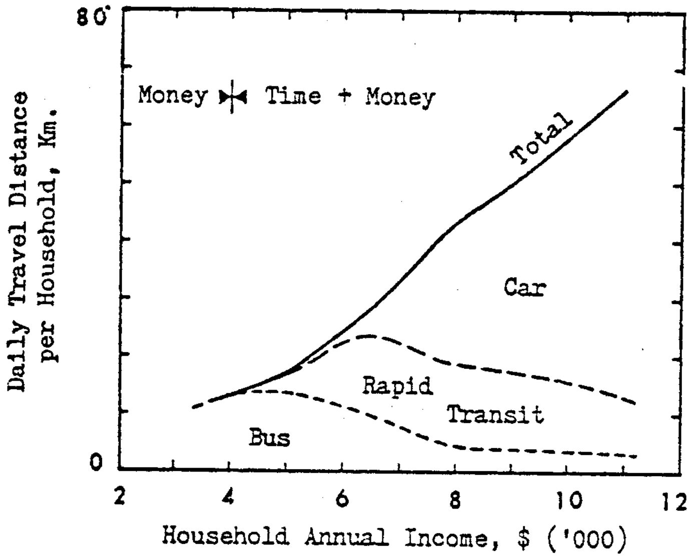

Figure 4 : Estimated Maximum Daily Travel Distance per Average Household, by Three Modes, Under the Travel Time and Money Budgets and their Constraining Ranges, vs. Household Annual Income, Based on Travel Budgets and Unit Costs in Washington, D.C. 1968

maximize their travel benefits. However, at a bifurcation point, when a mode is dropped from the choice-set, a household has to make a long-term (slow) decision, of whether to drop the use of such a mode (e.g., a car) and decrease the amount of travel, or increase the travel budget/s in order to keep the previous amount of travel.

This problem is made more clear when increasing (or decreasing) the 1968 unit costs of travel. Appendix 2 and Figure 5 show the results of such changes in unit costs for representative households with annual 1968 incomes of $ 6,000 and $ 5,000. As can be seen, the critical bifurcation point for the former household is reached when costs of travel increase by about 95 percent, while the latter household reaches the critical bifurcation point when travel costs increase by only 35 percent. Figure 5 also shows that if the travel budgets (the money budget in this case) are not changed, there is a sudden drop in the total daily travel distance per household.

It may, therefore, be concluded that mode choice is not necessarily a continuous function; continuous increases in travel costs may result in sudden changes in long-term mode choice at critical bifurcation points, which may differ for various household types. Thus, it appears that conventional, continuous, mode-choice models, calibrated to observed choices of single trips, may not be able to express the travel behavior phenomena already discussed.

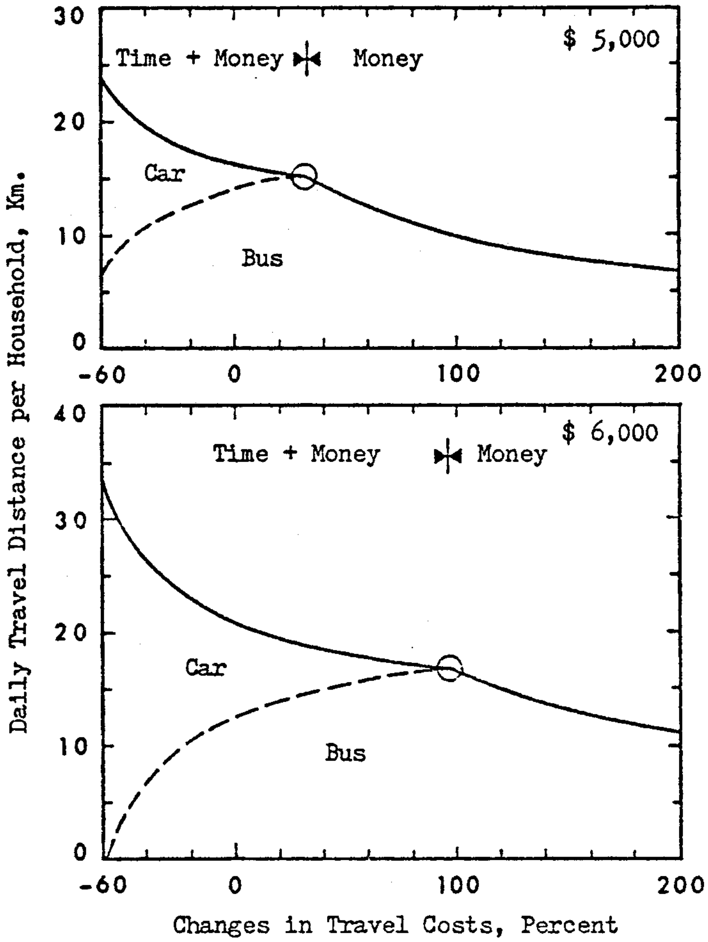

Figure 5 : Estimated Maximum Daily Travel Distance per Average Household, by Two Modes, Under the Travel Time and Money Budgets and their Constraining Ranges, vs. Changes in Travel Costs, for Two Income Levels, Based on Travel Budgets and Unit Costs in Washington, D.C. 1968

While in certain instances the problem of discontinuities may not be serious for aggregate models, which focus on aggregate travel behavior of groups of households, it becomes a serious problem for disaggregate behavioral models, which deal with individual households' travel behavior.

IV. THEORY OF BIFURCATION APPLIED TO MODE CHOICE

1. In the previous section we provided some demonstrations for bifurcation of individual mode choice behavior. We now turn to some theoretical issues behind such phenomena. A technical paper by the authors [5] presents a formal theory of sudden jumps in mode choice behavior for average daily total travel distance by a household/traveler. It is based on a utility of travel maximizing choice model, stemming from standard microeconomic analysis. According to this approach, the driving force for deriving the individual's equilibrium amount of distance traveled by a mode depends on the objective function employed by the traveler.

An objective function is used which depicts the "utility of Travel" notion, under the restriction that such utility is directly independent of any production/consumption activities. This objective function employs as arguments the distances traveled by mode. For fixed quantities of total travel time and money resources spent on travel, a higher utility of travel is always associated with a higher total individual utility (that accounts for production and consumption activities as well); this is implied in [5] and argued in [6]. It is argued that distances traveled are intrinsically associated with access to opportunities to consume and/or to produce, and that increases in travel distances always imply increases in access to opportunities. Given a spatial distribution of opportunities such that increases in distance traveled always results in increases in accessibility, there is a positive utility index associated with distance traveled by mode. However, everything else being equal, an individual would desire to reach the same quantity of opportunities traveling at lesser distances. This is the case because there is an inherent disutility associated with the intermediate nature of the travel activity. Thus, the overall travel utility is a composite of a positive utility of travel distance function and a negative utility due to traveling per se. A continuous distribution of opportunities over an isotropic plane would correspond to a continuous overall utility function.

The model uses a separable utility of travel objective function [5]:

$$\text{Max } U = \sum_{i \in I} (A_i d_i^4 - B_i d_i^2 + D_i d_i)$$

where: U - utility of travel;
d_i - daily travel distance per household/traveler by mode i;
A_i, B_i, D_i - coefficients associated with mode i;

such that the marginal rate of substitution between two modes is independent of the distance traveled by any other mode. In measuring the positive utility of distance traveled this may be a strong assumption if more than two modes are frequently used in each individual trip. This may not be a strong assumption when capturing the disutility of travel. From a mathematical standpoint the effect of a more complicated utility function poses no severe limitation on the qualitative results that can be obtained by employing such a utility function. Although at first glance it looks as if it is restricted, nonetheless, it possesses a number of general properties that are shown to have significant bearing on the phenomena mentioned above regarding individual mode-choice behavior. For one, it is capable of eliminating the strict concavity assumption found in conventional utility maximizing problems in economic analysis, by allowing increases/decreases over different ranges of total distances traveled by any mode over the value of the utility function given specific values in the parameters of the problem. It seems for certain modes never to be used without the utility of travel to vanish (a result not possible to obtain with Cobb-Douglas utility functions); and it allows for multiple solutions in the equilibrium total travel distances by mode for a given set of values in the problem parameters. This last requirement is critical in studying jumps in modal choice behavior and bifurcations in the dynamics of individual choices.

2. According to the simplified formulation of UMOT, a traveler is operating on the average day under two resource constraints:

$$\sum_i d_i/s_i \leq T$$

$$\sum_i d_i c_i \leq C$$

where: index i stands for mode ($i \in I$); d_i is total distance traveled by mode i; s_i is the average speed of mode i facing the specific traveler; c_i is the average money cost facing the traveler per unit distance traveled; T is the total travel time and C is the total travel money cost the traveler is willing to spend on the average per day.

From the necessary and sufficient conditions for individual equilibrium the model shows that a mode will not be employed by a traveler if its marginal travel disutility is positive, i.e., when the marginal utility of the equilibrium distance traveled by such mode is less than the sum of its time and money utilities at the margin. On the other hand, if a mode is utilized at equilibrium by a traveler then its marginal disutility is zero. Furthermore, the model states that at equilibrium the marginal utility of distance traveled by any used mode must be positive and equal to the sum of the time and money resources utilized for that mode at the margin.

The model's main aim is to show that there is a sudden change in the equilibrium value of the total distance traveled by any mode for slight perturbations of any parameter, including travel costs. The value of the parameter $\bar{c}_i$ that would produce discontinuities in modal choice behavior (either drastic increase or decrease in travel distance) is obtained by the model, and is given by:

$$\frac{\partial U}{\partial d_o} = (B_i \sqrt{3B_i} - D_i) \{ \bar{c}_i - (\gamma_i - \bar{c}_i)(1 - \bar{c}_i s_i)/(1 - s_i)\}^{-1}$$

where: $\partial U/\partial d_o$ is the marginal utility of a specific mode used as a reference (say walking) at equilibrium, $\bar{c}_i$ is the critical travel cost for mode i; γ_i is the marginal rate of substitution between mode i and the reference mode; and s_i is the speed of mode i. The detailed derivation of the above expression is found in [5].

Assuming that travel costs increase over time due to continuous increases in energy pricing, affecting differentially various modes due to different energy intensive technology employed, the model provides the length of time needed for the drastic change to occur.

The above results were obtained when income, and the upper bounds of travel time and money spent, were held constant over time. Fluctuations of these parameters between certain upper and lower bounds may further result in other bifurcation points for each individual traveler.

V. TRAVEL AND URBAN STRUCTURE

1. It was shown in the above sections how continuous changes in travel costs may result in discontinuous travel behavior of households/travelers, and how such discontinuities can be expressed mathematically by theories of bifurcation.

It is to be noted at this stage that different households, covering a wide range of incomes (and tastes, preferences, and other nonquantifiable attributes) may reach their critical travel (bifurcation) point at different travel cost increases and, hence, the discrete changes of many single households and travelers may result in practically continuous observed effects on travel behavior in a city.

It is also possible that single households may "soften" their critical (bifurcation) point by using various combinations of intermediate solutions, such as (i) delaying the increase in the unit cost of car travel by delaying the replacement of their aged car to a new car, or changing to a smaller car, while still traveling the same daily distance as before within their travel budgets, and (ii) increasing the travel money budget. Such solutions may transform a potentially sudden change in travel behavior into a more gradual and continuous change in a city, even for households within a given socioeconomic group. Hence, it may be argued that neither sudden changes in travel behavior nor in urban structure are to be expected, since they are not observed (and, therefore, are not represented in the calibrated models).

While this may be so on the basis of observed aggregate data, it is also evident from the analysis in the previous sections that continuous increases in travel costs are expected to force households to change their travel behavior in a significant way at some critical points along the increases in travel costs if the travel budgets are to be kept stable (e.g., Figure 5). From this viewpoint, there is an equivalence between the case where there is a sudden drop in travel distance within the stable travel budgets, and the case where there is a sudden increase in the travel money budget in order to travel the same distance, by the same modes, as before. Thus, even though aggregated observations of many households may mask such individual bifurcation points, it is evident that a long-term decision must be made by a household at a certain point, which may affect the whole economy. For instance, delaying the replacement of cars will affect the automotive industry, an effect which certainly is observed at an aggregate level. Similarly, increasing the travel money budget will affect all other money budgets, resulting in direct and indirect effects spreading to the whole economy which, once again, are evident at an aggregate level. Hence, a truly behavioral travel demand model has to express explicitly such critical points in the decision process of individual households and their travelers.

2. It was already shown that there are strong relationships between the spatial distributions of residences and jobs in urban areas and the trip distance [7], as well as between the trip rate per traveler and the proportions of trip purposes [8]. It is evident, therefore, that changes in travel behavior, such as decreases in the total daily travel distance per traveler/household, and/or transfer between modes, are liable to affect both the trip distance and the trip rate. Such possible changes in travel are expected to generate forces that can change urban structure; e.g., residences will tend to gravitate back towards the city center, and/or jobs will tend to spread towards the residences. While both effects can take place simultaneously, the net effect will depend on the relaxation (reaction) times of the urban structure components, such as the rates of residence and job relocations.

   If the relaxation time of urban structure is shorter than the rate at which travel behavior changes, urban structure may adjust itself to the new conditions without undue difficulties, and reach a new equilibrium condition. However, if travel behavior is expected to change, due to rapid increases in travel costs, at a faster rate than the relaxation time of urban structure, disequilibrium conditions may develop in a city. The point to note is that such disequilibrium conditions, even at an aggregate level, may develop explosively after the rate of increase of travel costs - and the resulting changes in travel behavior - exceed the relaxation time of urban structure. It appears, therefore, that a principal link between the relatively sudden bifurcation of travel behavior of individual households and the relatively sudden bifurcation of urban development and activity levels, is the relaxation times of the components of urban structure.

Unfortunately, not much is known about relaxation times; they were not regarded as crucial for the calibration and application of conventional travel demand and urban structure models as long as observed conditions - and the assumptions underlying such models - dealt with slow changes in urban development under relatively stable conditions. It appears, however, that we are approaching rapidly a critical situation where more knowledge about relaxation times of urban structure components may decide our ability to develop and apply reliable models for the prediction of the future of our cities.

3. In conclusion, the above explorations suggest that future research in the fields of urban travel behavior and urban structure concentrate on three principal subjects:

   (a) Development of dynamic travel demand models that can express explicitly bifurcation points in travel behavior of individual households under the constraints of travel budgets.

   (b) Development of dynamic urban structure models that can express explicitly bifurcation points in urban development under conditions of varying relaxation times.

   (c) Unifying both models, micro and macro, within the urban system, with special attention given to the feedback process between the two, and to possible disequilibrium conditions between travel demand and urban structure.

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REFERENCES

1. Rashevsky, N. (1951). Mathematical Biology of Social Behavior. The University of Chicago Press.

2. Nicolis, G. and Prigogine, I. (1977). Self Organization in Non-Equilibrium Systems. John Wiley & Sons.

3. Allen, P.M. et al. (1978). The Dynamics of Urban Evolution. Prepared for the U.S. Department of Transportation, Research and Special Programs Administration (RSPA). Final Report, October, No. DOT-TSC-RSPA-78-20, I & II.

4. Zahavi, Y. (1979). The UMOT Project. Prepared for the U.S. Department of Transportation, RSPA, and the Fed. Rep. of Germany Ministry of Transport. Final Report, August, No. DOT-RSPA-DPB-20-79-3.

5. Dendrinos, D.S. and Zahavi, Y. (1979). Critical Points in Dynamic Mode-Choice Behavior. U.S. Department of Transportation, RSPA, Office of Systems Engineering. December (Mimeo.)

6. Dendrinos, D.S. (1979). Elaboration on a New Modal Choice Model. U.S. Department of Transportation, RSPA, Office of Systems Engineering (Mimeo.)

7. Zahavi, Y. (1976). The Effects of Transportation Systems on the Spatial Distributions of Population and Jobs. Presented at the Joint National Meeting, ORSA/TIMS, Miami, November 1976.

8. Zahavi, Y. (1979/80). The UMOT/Urban Interactions. Prepared for the U.S. Department of Transportation, RSPA (in progress).

Appendix 1

Summary of Estimated Daily Travel Distance per Average Household, by Income, Under the Travel Time and Money Budgets and Unit Costs, Washington, D.C. 1968

Household Annual Income,$		4,000	5,000	6,000	7,000	8,000	9,000	10,000	11,000
Money Budget, M , $		0.51	0.75	1.24	2.01	2.82	3.17	3.53	3.88
Time Budget, T , min.		121.2	121.2	125.4	132.0	137.4	144.6	151.8	157.8
Unit Cost, c , $	Car	0.104	0.096	0.092	0.081	0.075	0.068	0.064	0.060
	Bus	0.0373	0.0373	0.0373	0.0373	0.0373	0.0373	0.0373	0.0373
	R.T.	0.056	0.056	0.056	0.056	0.056	0.056	0.056	0.056
t , min.	Car	4.44	4.00	3.75	3.16	2.86	2.50	2.31	2.14
	Bus	8.89	8.00	7.50	6.32	5.71	5.00	4.62	4.29
	R.T.	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
Daily Travel Distance within the:									
Money Budget	Car	4.90	7.81	13.48	24.81	37.60	46.62	55.16	64.67
	Bus	13.67	20.11	33.24	53.89	75.60	84.99	94.64	104.02
	R.T.	9.11	13.39	22.14	35.89	50.36	56.61	63.04	69.29
Time Budget	Car	27.30	30.30	33.44	41.77	48.04	57.84	65.71	73.74
	Bus	13.63	15.15	16.72	20.89	24.06	28.92	32.86	36.78
	R.T.	40.40	40.40	41.80	44.00	45.80	48.20	50.60	52.60
Daily Travel Distance by Mode:									
Car/Bus	Car	0.02	2.39	8.40	19.74	34.14	42.38	50.81	60.59
	Bus	13.62	13.95	12.52	11.02	6.97	7.73	7.45	6.56
	Total	13.64	16.34	20.92	30.76	41.11	50.11	58.26	67.15
Car/R.T.	Car	-	-	-	-	11.81	22.07	33.35	46.60
	R.T.	9.11	13.39	22.14	35.89	34.54	29.81	24.92	19.36
	Total	9.11	13.39	22.14	35.89	46.35	51.88	58.27	65.96
Bus/R.T.	Bus	13.62	12.67	10.72	5.63	-	-	-	-
	R.T.	0.03	4.95	15.00	32.14	45.80	48.20	50.60	52.60
	Total	13.65	17.62	25.72	37.77	45.80	48.20	50.60	52.60
Car/Bus/R.T.	Car	-	0.99	1.87	11.25	24.82	33.23	42.47	53.52
	Bus	13.62	13.69	11.12	8.70	4.06	4.25	3.88	3.25
	R.T.	0.03	2.57	11.67	13.83	14.42	13.43	11.94	9.75
	Total	13.65	17.25	24.66	33.78	43.30	50.91	58.[illegible]9	66.52

Appendix 2

Summary of Estimated Daily Travel Distance by Car and Bus per Household, for Two Income Levels, Under the Travel Time and Money Budgets and Changes in Travel Unit Costs, Washington, D.C. 1968

Household Annual Income, $	5,000			6,000		
Money Budget, M, $	0.75			1.24		
Time Budget, T, min.	121.2			125.4		
Unit Cost, c, $ Car	0.096			0.092		
Bus	0.0373			0.0373		
t, min. Car	4.00			3.75		
Bus	8.00			7.50		
Daily Travel Distance by Mode, km., after Changing Money Unit Costs by:	Car	Bus	Total	Car	Bus	Total
0	2.39	13.95	16.34	8.40	12.52	20.92
+ 20	0.77	14.76	15.53	5.57	13.93	19.50
+ 40	-	14.42	14.42	3.60	14.92	18.52
+ 60	-	12.50	12.50	2.02	15.71	17.73
+ 80	-	11.19	11.19	0.90	16.27	17.17
+ 100	-	10.00	10.00	-	16.53	16.53
+ 120	-	9.15	9.15	-	15.12	15.12
+ 140	-	8.33	8.33	-	13.78	13.78
+ 160	-	7.73	7.73	-	12.78	12.78
+ 180	-	7.21	7.21	-	11.92	11.92
+ 200	-	6.70	6.70	-	11.07	11.07
- 20	4.82	12.74	17.56	12.64	10.40	23.04
- 40	8.85	10.73	19.58	19.67	6.89	26.56
- 60	16.94	6.68	23.62	33.70	-	33.70